Available online at www.sciencedirect.com
ScienceDirect
Physica D 237 (2008) 2127–2131
PHYSICA D
www.elsevier.com/locate/physd

Rogue waves in oceanic turbulence

Francesco Fedele*

School of Civil & Environmental Engineering, Georgia Institute of Technology, Savannah Georgia, USA

Available online 31 January 2008

Abstract

A stochastic model of wave groups is presented to explain the occurrence of exceptionally large waves, usually referred to as rogue waves. The model leads to the description of the non-Gaussian statistics of large waves in oceanic turbulence and to a new asymptotic distribution of their crest heights in a form that generalizes the Tayfun model. The new model explains the unusually large crests observed in flume experiments of narrow-band waves. However, comparisons with realistic oceanic measurements gathered in the North Sea during an intense storm indicate that the generalized model agrees with the original Tayfun distribution.

PACS: 92.10.Hm; 91.30.Fn

Keywords: Rogue wave; Weak wave turbulence; Wave group; Non-Gaussian; Crest statistics

1. Introduction

Rogue waves are extreme events with potentially devastating effects on offshore structures and ships. A rogue wave observed at the Draupner platform in the North Sea during a storm in January, 1995 provides evidence that such waves can occur in the open ocean. Theoretical models offer various physical mechanisms that can produce such focusing of wave energy in a small area of the ocean. When nonlinearities are negligible, ocean waves are usually modeled as Gaussian seas, as a linear superimposition of a large number of elementary waves with amplitudes related to a given spectrum and random phases. In this case, large waves occur due to the dynamics of a large stochastic wave group evolving linearly in accordance with both the Slepian model [1] and the theory of quasi-determinism of Boccotti [2]. Moreover, crests and troughs are both Rayleigh-distributed. If second-order nonlinearities are dominant, then the sea surface displays sharper narrower crests and shallower more rounded troughs. As a result, the skewness of surface elevations is positive [3], and wave crests are distributed according to the Tayfun model [4–7]. If, however, elementary waves also exchange energy nonlinearly via third-order four-wave resonances, narrow-band wave trains can undergo intense modulational instability enhancing the occurrence of larger waves [5,8] and, as a result, the distribution of crest heights can deviate from the Tayfun model. This is confirmed by both the wave-flume experiments in [6] and the numerical simulations of the Dysthe equation [7], a special case of the Zakharov equation [9] governing the dynamics of weakly nonlinear water waves. The unusually large wave crests observed in both the latter experiments and simulations are explained reasonably well by a Gram–Charlier approximation of the crest distribution recently proposed in [5]. This model stems from the general Hermite series expansion of random variables [3], and it relates to the physics of ocean waves only through various statistics such as the skewness and kurtosis of surface displacements. Could such type of Gram–Charlier models for crests proceed directly from the basic equations governing the ocean dynamics without assuming a priori that the associated statistical structure is in the form of a Gram–Charlier expansion in Hermite polynomials ? This paper will explore this query by formulating a new stochastic model of wave groups, describing the non-Gaussian statistics of large waves under conditions referred to as oceanic or wave turbulence (WT). The latter state defines the chaotic behavior of a sea of weakly nonlinear-coupled dispersive wave trains evolving in accordance with the Zakharov equation [9]. An

* Tel.: +1 902 966 6785.
E-mail address: francesco.fedele@gtsav.gatech.edu.

0167-2789/$ - see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.physd.2008.01.022

initial Gaussian field is weakly modulated as nonlinearities develop in time, leading to intermittency in the turbulent signal due to the formation of sparse but intense coherent structures. Large wave crests observed during these localized events may explain the occurrence of rogue waves in open ocean. By exploiting the weak nature of the nonlinear interactions of $O(\mu^2)$, with μ defined as a small parameter for wave steepness, large crests are identified as those waves riding on top of large groups. In fact, for time scales much larger than the typical wave period T_L, but much less than the nonlinear time scale $T_{NL} \sim O(\mu^{-2})$, waves traveling in groups evolve mainly due to the faster non-resonant second-order interactions while the slower third-order resonant interactions modify and intensify their amplitudes. Thus, the initial deviations from Gaussianity observed in the statistical structure of large waves are revealed before turbulence becomes strong and thus the WT theory breaks down.

Herein, the WT theory is briefly reviewed first. Then, some salient features of the concept of stochastic wave groups relevant to WT are discussed, leading to a generalization of the Tayfun model for the statistical distribution of crest heights over large waves. Finally, comparisons with the lab data, numerical simulations and wave measurements collected in the North Sea are presented.

2. Oceanic turbulence

Consider weakly nonlinear random waves propagating in water of uniform depth d in accordance with the Zakharov equation for WT [9]. Define $\mathbf{x} = (x, y)$ as the horizontal position vector on a plane coincident with the water mean level, t the time, $\mathbf{k}$ as the horizontal wave-number vector, and ω is the angular frequency related to k via $gk \tanh kd = \omega^2$, with $k = |\mathbf{k}|$. Drawing upon [11], the sea surface displacement ζ from the mean sea level is given, correct to $O(\mu^2)$, by

$$\zeta = \zeta_1 + \zeta_2, \tag{1}$$

where the component ζ_1, that accounts for four-wave resonant interactions, is given by

$$\zeta_1 = \int b_1(t) \mathrm{e}^{\mathrm{i}(\boldsymbol{\theta}_1 - \Omega_1 t)} \mathrm{d}\mathbf{k}_1 + c.c. \tag{2}$$

with $\boldsymbol{\theta}_1 = \mathbf{k}_1 \cdot \mathbf{x} - \omega_1 t$ and $b_1(t) = b(\mathbf{k}_1, t)$ a complex amplitude whose perturbation expansion in small μ is given, correct to $O(\mu^2)$, by [11]

$$b_1(t) = B_1(1 + \mathrm{i}\mu^2 \Omega_1 t) - 2\mu^2 g \left[\mathcal{G}(t; B) - \mathcal{G}(0; B)\right], \tag{3}$$

where

$$\Omega_1 = 2\omega_1 \int W_{12}^{12} |A_2|^2 \, d\mathbf{k}_2$$

is the renormalization frequency arising from the nonlinear frequency shift due to self-interactions, and

$$\mathcal{G}(t; B) = \int W_{34}^{12} \sqrt{\frac{\omega_1}{\omega_2\omega_3\omega_4}} \bar{B}_2 B_3 B_4 \delta_{34}^{12} \frac{\exp\left(-i\omega_{34}^{12} t\right)}{\omega_{34}^{12}} \mathrm{d}\mathbf{k}_{234},$$

is a function of the initial amplitudes $B_1 = B(\mathbf{k}_1)$ at $t = 0$, W_{34}^{12} is the four-wave interaction kernel, $\omega_{34}^{12} = \omega_1 + \omega_2 - \omega_3 - \omega_4$, $\delta_{34}^{12} = \delta(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3 - \mathbf{k}_4)$, and $\bar{B}$ is the complex conjugate of B. The correction ζ_2 due to non-resonant interactions is given by

$$\zeta_2 = \int b_1 b_2 \left[A_{12}^{+} \mathrm{e}^{\mathrm{i}(\boldsymbol{\theta}_1 + \boldsymbol{\theta}_2)} + A_{12}^{-} \mathrm{e}^{\mathrm{i}(\boldsymbol{\theta}_1 - \boldsymbol{\theta}_2)}\right] \mathrm{d}\mathbf{k}_{12} + c.c. \tag{4}$$

where $A_{12}^{\pm} = A^{\pm}(\mathbf{k}_1, \mathbf{k}_2)$ are interaction coefficients [10]. Clearly $\langle \zeta \rangle = 0$ and the variance $\langle \zeta^2 \rangle = \sigma^2$, where $\langle \cdot \rangle$ stands for expected value.

3. Large crests in Gaussian seas

Neglect both resonant and non-resonant interactions so that ζ_1 is Gaussian. Further, assume that a large wave crest of amplitude h is recorded at $\mathbf{x} = \mathbf{x}_0$ and $t = t_0$. Boccotti [2] shows that as $h/\sigma \to \infty$, with probability approaching 1, the large crest occurs when a well-defined wave group ζ_c passes through $\mathbf{x}_0$. The surface displacement of ζ_c around $\mathbf{x} = \mathbf{x}_0 + \mathbf{X}$ and $t = t_0 + T$ is asymptotically described by the following conditional process

$$\zeta_c = \{\zeta_1(\mathbf{X}, T) | \zeta_1(\mathbf{0}, 0) = h\} = \zeta_{\mathrm{det}} + \mathcal{R}_\zeta, \tag{5}$$

as the sum of a deterministic part ζ_{det} previously derived in [1,2], and given by

$$\zeta_{\mathrm{det}} = \langle \zeta_1(\mathbf{X}, T) | \zeta_1(\mathbf{0}, 0) = h \rangle = h \frac{\Psi}{\sigma^2},$$

and a random residual $\mathcal{R}_\zeta$ that can be explicitly expressed as (see [12] and also Appendix for details)

$$\mathcal{R}_\zeta(\mathbf{X}, T) = \frac{\Delta}{\sigma^2} \frac{-\psi^* \; \Psi(\mathbf{X}, T) + \Psi(\mathbf{X}, T - T^*)}{1 - \psi^{*2}} + O(h^{-1}),$$

where Δ is a random variable of $O(h^0)$, and Ψ is the space–time covariance of ζ_1 given by

$$\Psi(\mathbf{X}, T) = \int S_1 \cos(\mathbf{k}_1 \cdot \boldsymbol{X} - \omega_1 T) \mathrm{d}\mathbf{k}_1,$$

with $S(\mathbf{k}_1) =$ the wave-spectral density with bandwidth ν, and $\psi^* \equiv \psi(T^*)/\psi(0)$ with T^* being the abscissa of the first local minimum of the time covariance $\psi(T) = \Psi(\mathbf{0}, T)$. Since h and Δ are random variables, ζ_c identifies a *stochastic wave group* which evolves linearly through a wave background represented by the residual $\mathcal{R}_\zeta$. The largest crest occurs as waves, growing from the tail of the group, reach its apex [2]. The dimensionless variables $\xi = h/\sigma$ and $\tilde{\Delta} = \Delta/\sigma$ are stochastically independent, as $\xi \to \infty$. Moreover, ξ is Rayleigh-distributed and $\tilde{\Delta}$ is Gaussian with zero mean and variance $1 - \psi^{*2}$. In the following, it will be useful to express ζ_c in the form

$$\zeta_c = \int \tilde{B}_1 \mathrm{e}^{\mathrm{i}\boldsymbol{\theta}_1} \mathrm{d}\mathbf{k}_1 + c.c. \tag{6}$$

where

$$\tilde{B}_1 = \left(h - \Delta \frac{-\psi^* + \mathrm{e}^{\mathrm{i}\omega_1 T^*}}{1 - \psi^{*2}}\right) \frac{S_1}{2\sigma^2} \mathrm{e}^{-\mathrm{i}(\mathbf{k}_1 \cdot \mathbf{x}_0 - \omega_1 t_0)}, \tag{7}$$

with $S_1 = S(\mathbf{k}_1)$. Hereafter, the concept of stochastic wave groups is exploited to explain the occurrence of large waves and the associated crest statistics in WT.

4. Large crests in oceanic turbulence

Consider the nonlinear surface ζ. Because of both the fast non-resonant and slow resonant interactions, crest statistics deviate from being Gaussian. Such deviations can be quantified by drawing upon [13,14]. So, assuming that a large crest of amplitude h_{nl} is recorded at $\mathbf{x} = \mathbf{x}_0$ and $t = t_0$, ζ surrounding that crest locally around $\mathbf{x} = \mathbf{x}_0 + \mathbf{X}$ and $t = t_0 + T$ is given by the nonlinear conditional process

$$\zeta_{nc} = \{\zeta(\mathbf{X}, T) | \zeta(\mathbf{0}, 0) = h_{nl}\}. \tag{8}$$

If the waves were Gaussian, ζ_{nc} would be identical to the wave group ζ_c in (5). For nonlinear waves, does ζ_{nc} still represent a group forming a large crest with amplitude h_{nl}? The answer to this question is given by exploiting the weakly nature of the nonlinear interactions of ζ. First, ignore four-wave resonances in (1). Then, ζ_1 is Gaussian and ζ is homogeneous in space and time, but non-Gaussian. Under these conditions, the crest statistics deviate from the Rayleigh distribution, but they are well described by the Tayfun distribution [4,5,12]. For long-crested narrow-band waves in deep water, as the spectral bandwidth $\nu \to 0$, ζ assumes the simple form [4,5]

$$\zeta = \zeta_1 + \frac{\mu}{2\sigma}\left(\zeta_1^2 - \hat{\zeta}_1^2\right) + O(\nu), \tag{9}$$

where the component ζ_2 is explicitly identified in terms of ζ_1, $\mu = \lambda_3/3$ is related to the skewness coefficient $\lambda_3 = \langle \zeta^3 \rangle / \sigma^3$, and $\hat{\zeta}_1$ denotes the Hilbert transform of ζ_1 with respect to time. From (9) it is clear that the component ζ_2 is phase-coupled to the extremes of the Gaussian ζ_1. So, a large crest of ζ with amplitude h_{nl} occurs simultaneously when ζ_1 itself is at a large crest with an amplitude, say h [14]. Thus, the conditional process (8) is equivalent to the simpler process

$$\zeta_{nc} = \{\zeta | \zeta_1 = \zeta_c\}, \tag{10}$$

which explicitly follows, by replacing ζ_1 in (9) with ζ_c of (6), as

$$\zeta_{nc} = \zeta_c + \frac{\mu}{2\sigma}\left(\zeta_c^2 - \hat{\zeta}_c^2\right). \tag{11}$$

The amplitude h_{nl} of the largest crest of ζ_{nc} occurs at $\mathbf{x} = \mathbf{x}_0$ and $t = t_0$, i.e. $\mathbf{X} = \mathbf{0}$ and $T = 0$, when $\zeta_c = h$ and $\hat{\zeta}_c = 0$, and it is given in the *Tayfun form* as

$$\xi_{\max} = \xi + \frac{\mu}{2}\xi^2, \tag{12}$$

where $\xi_{\max} = h_{nl}/\sigma$ [12]. Thus, the Tayfun (T) model for the exceedance of the crest height $\xi_{\max}$ readily follows from the Rayleigh distribution of ξ as [4]

$$\Pr\{\xi_{\max} > \lambda\} = \exp\left(-\frac{\xi_0^2}{2}\right), \tag{13}$$

where ξ_0 satisfies the quadratic equation

$$\xi_0 + \frac{\mu}{2}\xi_0^2 = \lambda. \tag{14}$$

For $T_L \ll t_0 \ll T_{NL} \sim O(\mu^{-2})$, third-order resonant interactions develop and the wave field becomes nonstationary in time but still homogeneous in space. Moreover, the crest statistics deviates from the Tayfun model because the latter is based on the particular non-resonant form (9) of the generic ζ in (1). The deviations from the second-order theory can be still quantified by exploiting the space–time evolution of wave groups. In fact, the new group ζ_{nc} in (10) arising from the four-wave resonances of narrow-band waves is given by

$$\zeta_{nc} = \zeta_d + \frac{\mu}{2\sigma}\left(\zeta_d^2 - \hat{\zeta}_d^2\right), \tag{15}$$

where ζ_d originates from the modulation of the group ζ_c in (6) from pure resonant interactions. An explicit expression for ζ_d stems from ζ_1 in (2) by replacing the initial values B_1 of the associated complex amplitude $b_1(t)$ in (3) with those values $\tilde{B}_1$ of ζ_c in (7), that is

$$\zeta_d = \int \mathrm{e}^{\mathrm{i}(\boldsymbol{\theta}_1 - \Omega_1 t)} \Big\{ \tilde{B}_1(1 + \mathrm{i}\mu^2 \Omega_1 t) - 2\mu^2 \Big[\mathcal{G}(t; \tilde{B}) - \mathcal{G}(0; \tilde{B})\Big]\Big\} \,\mathrm{d}\mathbf{k}_1 + c.c. \tag{16}$$

By a direct inspection of both (15) and (16) one can show that the nonlinear group ζ_{nc} still focuses at $\mathbf{x} = \mathbf{x}_0$ and $t = t_0$ for $t_0 \ll T_{NL}$ with the largest crest amplitude $\xi_{\max}$ given by

$$\xi_{\max} = \xi + \frac{\mu}{2}\xi^2 + \mathcal{I}(t_0)\xi^3 + \mathcal{A}(t_0)\xi^2\tilde{\Delta} + \mathcal{B}(t_0)\xi\tilde{\Delta}^2, \tag{17}$$

where $O(\tilde{\Delta}^3, \mu^3)$ terms have been neglected and the dependence on $\mathbf{x}_0$ drops out because the field is homogeneous in space but nonstationary in time. Moreover $\mathcal{I}$, $\mathcal{A}$ and $\mathcal{B}$ are multidimensional integrals in $(\mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)$ space. In particular,

$$\mathcal{I} = \int Q_{34}^{12} S_2 S_3 S_4 \,\mathrm{d}\mathbf{k}_{234},$$

with

$$Q_{34}^{12} = \frac{\mu^2 g}{2m_0^2} W_{34}^{12} \sqrt{\frac{\omega_1}{\omega_2\omega_3\omega_4}}\, \delta_{34}^{12} \frac{1 - \cos\left(\omega_{34}^{12} t_0\right)}{\omega_{34}^{12}},$$

and in the narrow-band limit, as $\nu \to 0$,

$$\mathcal{A} \approx O(\nu), \qquad \mathcal{B} = -3\mathcal{I}\Big/\left(1 - \psi^{*2}\right) + O(\nu).$$

Drawing upon [8], the coefficient $\mathcal{I}$ relates to the fourth-order cumulant $\lambda_{40} = \mu_4 - 3$ of the wave surface as $\lambda_{40} = 24\mathcal{I}$, μ_4 being the kurtosis. The probability of exceedance for the nonlinear wave-crest height $\xi_{\max}$ is given by

$$\Pr\{\xi_{\max} > \lambda\} = \int_{-\infty}^{\infty} \Pr\left\{\xi > \xi^*(\lambda) \,\middle|\, \tilde{\Delta}\right\} \, p_{\tilde{\Delta}} \mathrm{d}\tilde{\Delta}, \tag{18}$$

where $p_{\tilde{\Delta}}$ is the Gaussian density of $\tilde{\Delta}$, ξ is Rayleigh-distributed and its associated threshold ξ^* satisfies $\xi_{\max} = \lambda$ in (17). Further, correct to $O(\nu)$, ξ^* can be Taylor-expanded in terms of $\tilde{\Delta}$ starting from $\xi = \xi_0$ of (14) as

$$\xi^* = \xi_0 - \frac{\lambda_{40}}{24}\left(\xi_0^3 - \frac{3\xi_0\tilde{\Delta}^2}{1 - \psi^{*2}}\right) + O(\tilde{\Delta}^3).$$

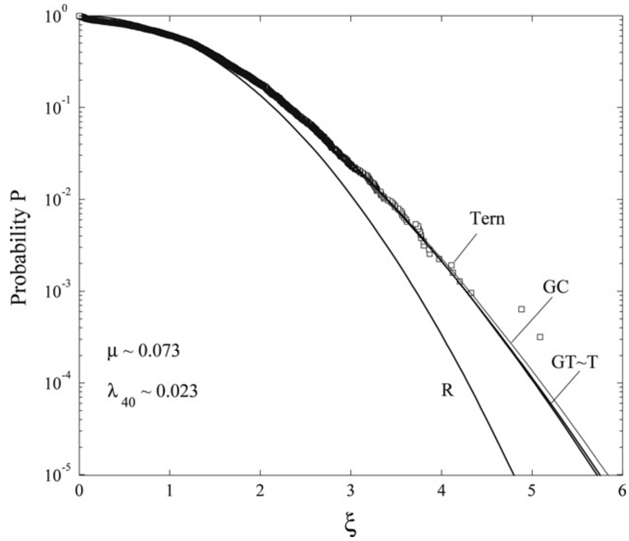

Fig. 1. Crest exceedances from Tern in comparison with the Tayfun, generalized Tayfun and Gram–Charlier models. Labels: R=Rayleigh, T = Tayfun (μ), GT= generalized Tayfun (μ, λ_{40}), GC= Gram–Charlier.

Ignoring terms of $O(\tilde{\Delta}^3)$ in the integration of (18) yields the probability of exceedance for $\xi_{\max}$ as

$$\Pr\{\xi_{\max} > \lambda\} = \exp\left(-\frac{1}{2}\xi_0^2\right)\left[1 + \frac{\lambda_{40}}{24}\lambda^2\left(\lambda^2 - 3\right)\right],$$

correct to $O(\mu^2)$. We shall refer to this asymptotic result, as the generalized Tayfun (GT) distribution, which is very similar to the Gram–Charlier (GC) approximation proposed in [5], viz.

$$\Pr\{\xi_{\max} > \lambda\} = \exp\left(-\frac{1}{2}\xi_0^2\right)\left[1 + \frac{\lambda_{40}}{24}\lambda^2\left(\lambda^2 - 4\right)\right].$$

Note that for directional broadband waves, wave-number quadruplets are in perfect resonance, i.e. $\omega_{34}^{12} = 0$, and the Tayfun model is recovered from both the GT and GC models since $\lambda_{40} = 0$.

5. Comparisons

The data to be considered for comparisons here comprise 9 h of measurements gathered during a severe storm in January, 1993 with a Marex radar from the Tern platform located in the northern North Sea in 167 m water depth. This data set is hereafter simply referred to as Tern. Tern represents storm seas under fairly steady conditions with broadband spectra characterized with $\sigma = 3.024$ m, spectral bandwidth $\nu = 0.629$ and $\lambda_3 = 0.174$. A stable estimate of the steepness μ in terms of spectral properties is given by $\mu_a = \mu_m\left(1 - \nu + \nu^2\right)$ [12]. In Fig. 1, the empirical distribution from Tern is compared with the T ($\mu \simeq \mu_a = 0.073$), GT ($\mu \simeq \mu_a = 0.073$, $\lambda_{40} \simeq 0.023$) and GC models respectively. It is observed that both the GT and GC models do not appear to improve significantly the predictions derived from the simpler T model. For most practical applications, the differences between the models appear insignificant, falling within a band of 1%–2%. Consider now the case of unidirectional narrow-band waves. The trend of the experimental wave-flume data of Fig. 2 in [6] is reproduced and shown in Fig. 2 here together with the predictions based on GT, GC ($\mu \simeq 0.075$, $\lambda_{40} \simeq 0.80$) and T ($\mu \simeq 0.075$) models. The original T model tends to underestimate the data whereas both the GT and GC models appear to explain data qualitatively well. The latter models also describe well the crest-height distribution from Fig. 9 (case C) of [7] obtained from numerical simulations of the Dysthe equation, reproduced and shown in Fig. 3 in comparison with the GT, GC ($\mu \simeq 0.07$, $\lambda_{40} \simeq 0.40$) and T ($\mu \simeq 0.07$) models.

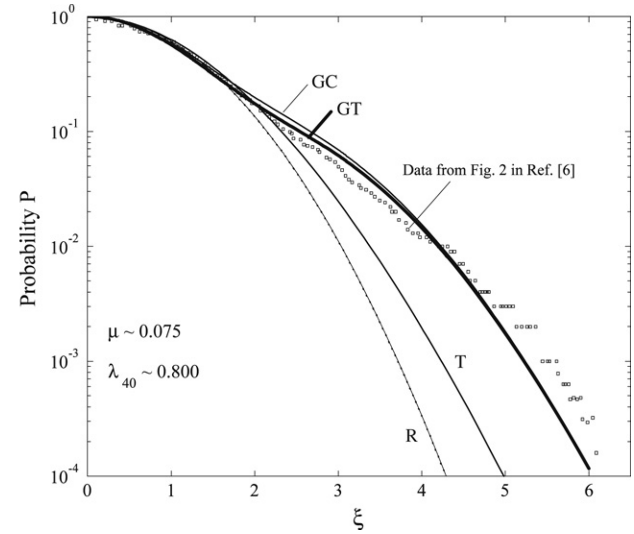

Fig. 2. Crest-height distribution from wave-flume experiments (Fig. 2 in [6]) in comparison with the Tayfun, generalized Tayfun and Gram–Charlier models. Labels are as for Fig. 1.

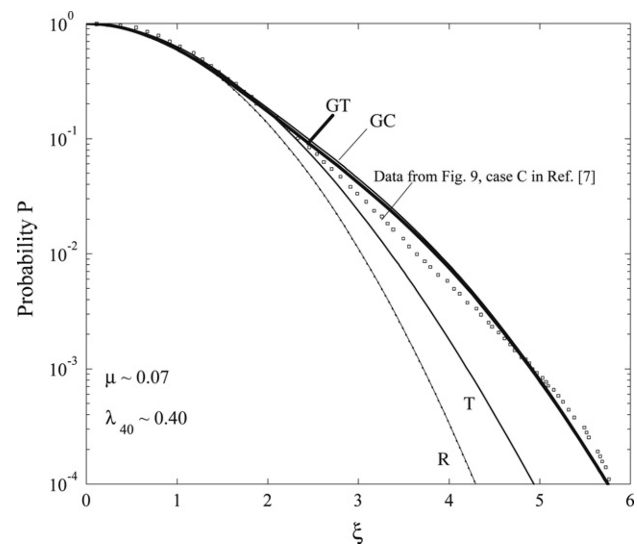

Fig. 3. Crest exceedances from numerical simulations (Fig. 9, case C in [7]) in comparison with the Tayfun, generalized Tayfun and Gram–Charlier models. Labels are as for Fig. 1.

6. Conclusions

A generalized Tayfun model for the statistics of crest heights over large waves in oceanic turbulence is proposed. The new crest model can explain the deviations from the Tayfun distribution observed in flume experiments of narrow-band waves. However, for realistic oceanic sea states the differences between the predictions of the new model and the Tayfun distribution appear negligible.

Acknowledgments

The author thanks M. Aziz Tayfun for useful comments and discussions and George Forristall for the data utilized in the paper.

Appendix

The wave profile $\eta_c(T)$ at $\mathbf{X}=\mathbf{0}$ is expressed in terms of an $O(h)$ contribution $\eta_{\text{det}}(T) = \zeta_{\text{det}}(\mathbf{0}, T)$ and the random residual $r(T) = \mathcal{R}_\zeta(\mathbf{0}, T)$ of $O(h^0)$ as

$$\eta_c(T) = \eta_{\text{det}}(T) + r(T),$$

where $\eta_{\text{det}}(T) = \zeta_{\text{det}}(\mathbf{0}, T) = h\psi(T)/\sigma^2$. Drawing upon [15], the effects of the residual $r(T)$ on η_c are now determined. Specifically, as $h/\sigma \to \infty$, with probability approaching 1, the surface profile locally near a large crest tends to assume the shape given by $\eta_{\text{det}}(T)$ [1,2]. The latter represents a wave profile with a crest of amplitude h at time $T = 0$ followed by the absolute minimum of amplitude $\eta_{\text{det}}(T^*)$ at $T = T^*$, with T^* being the abscissa of the first local minimum of $\psi(T)$. For large values of h, the wave trough of the profile $\eta_c(T)$ following the crest of amplitude h shall now occur at time $T = T^* + u$, with u being random. As $h/\sigma \to \infty$, a crest of amplitude h that occurs at $T = 0$, is followed after a time lag $T^* + u$ by a trough, and $\eta_c(T)$ and its first time derivative $\dot{\eta}_c(T)$ at $T = T^*$ attain values given, correct to $O\left(h^0\right)$, by

$$\eta_c(T^*) = \eta_{\text{det}}(T^*) + \Delta, \qquad \dot{\eta}_c(T^*) = -\ddot{\eta}_{\text{det}}(T^*)u.$$

For linear Gaussian functions, an approximation to $\eta_c(T)$ satisfying the preceding conditions exactly is given by

$$\eta_c(T) = \eta_{\text{det}}(T) + \frac{\Delta}{\sigma^2}\frac{-\psi(T)\psi(T^*)/\sigma^2 + \psi(T - T^*)}{1 - \psi(T^*)^2/\sigma^4},$$

where u drops out ignoring terms of $O(h^{-1})$. The straightforward extension of the above time formulation to the space–time domain leads to (5).

References

[1] G. Lindgren, Local maxima of Gaussian fields, Ark. Mat. 10 (1972) 195–218.
[2] P. Boccotti, Wave Mechanics for Ocean Engineering, Elsevier Science, Oxford, 2000.
[3] M.S. Longuet-Higgins, The effects of non-linearities on statistical distributions in the theory of sea waves, J. Fluid Mech. 17 (1963) 459–480.
[4] M.A. Tayfun, Narrow-band nonlinear sea waves, J. Geophys. Res. 85 (C3) (1980) 1548–1552.
[5] A. Tayfun, F. Fedele, Wave-height distributions and nonlinear effects, Ocean Eng. 34 (2007) 1631–1634.
[6] M. Onorato, A.M. Osborne, M. Serio, L. Cavaleri, C. Brandini, C.T. Stansberg, Extreme waves, modulational instability and second order theory: Wave flume experiments on irregular waves, Eur. J. Mech. B-Fluids 25 (2006) 586–601.
[7] H. Socquet-Juglard, K. Dysthe, K. Trulsen, H.E. Krogstad, J. Liu, Probability distributions of surface gravity waves during spectral changes, J. Fluid Mech. 542 (2005) 195–216.
[8] Peter A.E.M. Janssen, Nonlinear four-wave interactions and freak waves, J. Phys. Oceanogr. 33 (4) (2003) 863–884.
[9] V.E. Zakharov, Statistical theory of gravity and capillary waves on the surface of a finite-depth fluid, Eur. J. Mech. B-fluids 18 (3) (1999) 327–344.
[10] J.N. Sharma, R.G. Dean, Development and evaluation of a procedure for simulating a random directional second order sea surface and associated wave forces, Ocean Engineering Report no. 20, University of Delaware, 1979.
[11] Y. Choi, Y.V. Lvov, S. Nazarenko, B. Pokorni, Anomalous probability of large amplitudes in wave turbulence, Phys. Lett. A 339 (2005) 361–369.
[12] F. Fedele, M.A. Tayfun, On nonlinear wave groups and crest statistics, J. Fluid Mech. (under review).
[13] F. Fedele, Extreme events in nonlinear random seas, ASME J. Offshore Mech. Arc. Eng. 128 (1) (2006) 11–16.
[14] F. Fedele, F. Arena, Weakly nonlinear statistics of high non-linear random waves, Phys. Fluids 17 (1) (2005) 026601.
[15] P. Boccotti, On mechanics of irregular gravity waves, Atti Accad. Naz. Lincei Memorie VIII 19 (1989) 111–170.