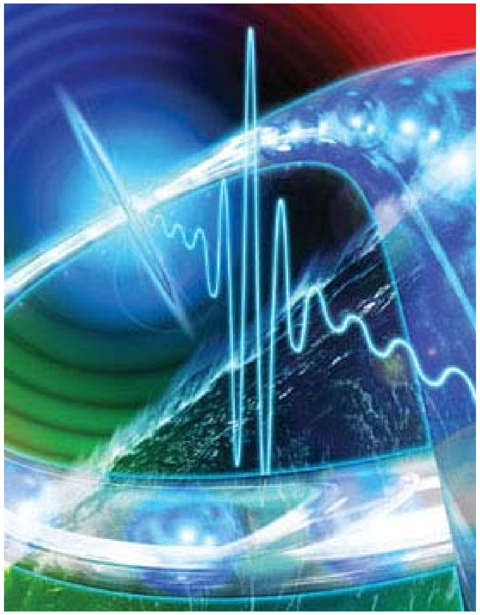
ROGUE WAVES IN OCEANIC TURBULENCE



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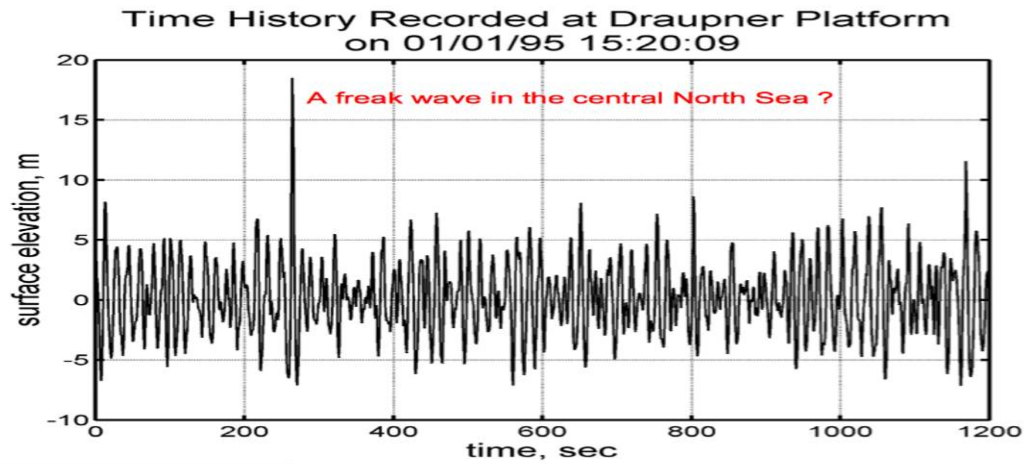
Rogue waves

Freak waves

DRAUPNER EVENT JANUARY 1995



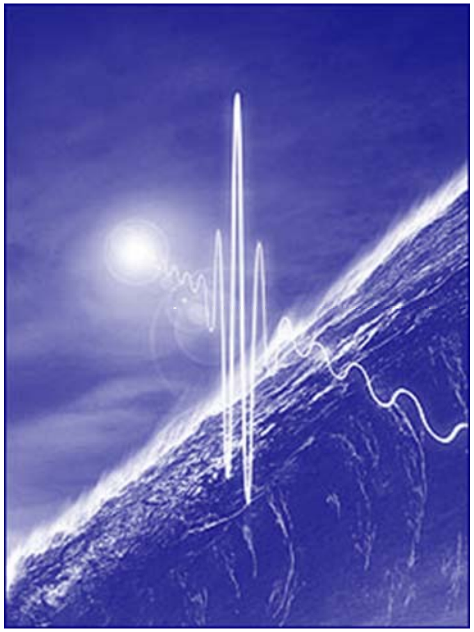
H_{max}=25.6 m !

Extremely rare event according to Gaussian model!!!

But they still occur in open ocean !

ROGUE WAVES

Rare events of a normal population
(*bad day at the tower*)
or
typical events of a special population ?
(*need of new physics*)



ANSWER hidden in the statistical structure of weakly nonlinear random waves

MAIN RESULT

Crest exceedance for weakly nonlinear random fields

$$\Pr[\mathbf{crest} > h] \propto \int_h^\infty \frac{S}{2\pi}\sqrt{\partial_{tt}F \cdot \partial_{ss}F}\,\frac{d^2 \Pr(\eta > z)}{dh^2}dz$$

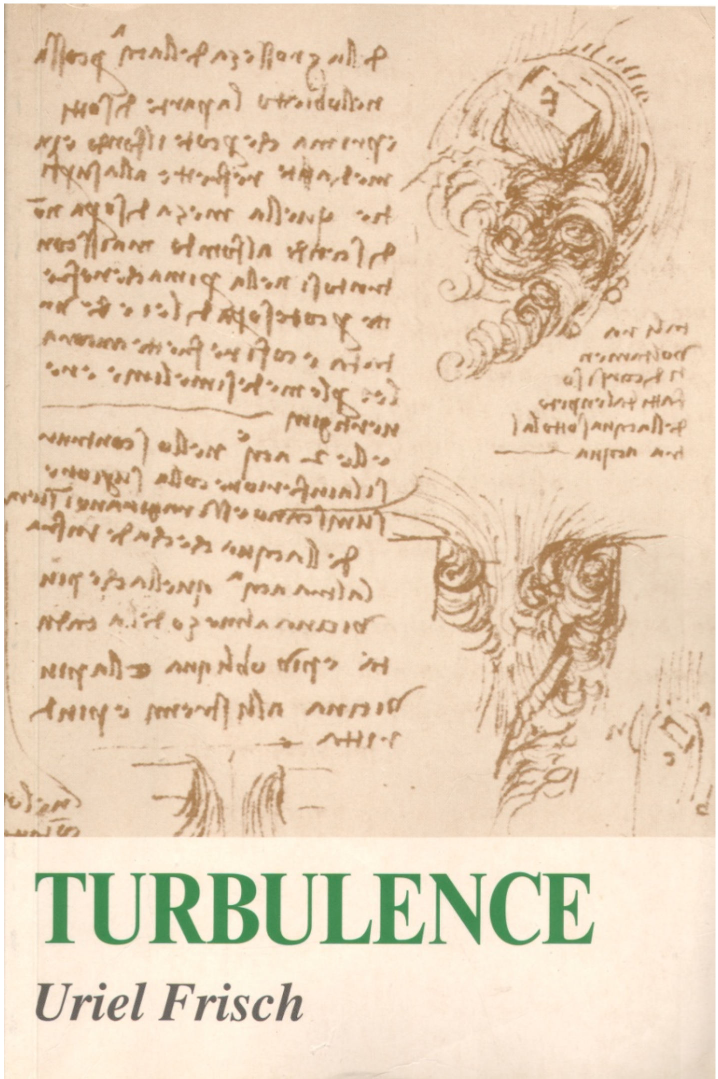
COROLLARY

the Generalized Tayfun distribution (GT) for the oceanic turbulence of Zakharov

$$\Pr(\mathbf{crest} > Z) = \exp\left[-\frac{\left(-1+\sqrt{1+2\mu Z}\right)^2}{2\mu^2}\right]\left[1+\frac{\lambda_{40}}{24}Z^2\left(Z^2-3\right)\right]$$

MY CONTRIBUTION

CONCEPT OF NONLINEAR STOCHASTIC WAVE GROUP
(Nonlinear Slepian model)



1.1 Turbulence and symmetries

In Chapter 41 of his *Lectures on Physics*, devoted to hydrodynamics and turbulence, Richard Feynman (1964) observes this:

Often, people in some unjustified fear of physics say you can't write an equation for life. Well, perhaps we can. As a matter of fact, we very possibly already have the equation to a sufficient approximation when we write the equation of quantum mechanics:

$$H\psi = -\frac{\hbar}{i}\frac{\partial \psi}{\partial t}. \qquad (1.1)$$

Of course, if we only had this equation, without detailed observation of biological phenomena, we would be unable to reconstruct them. Feynman believes, and this author shares his viewpoint, that an analogous situation prevails in *turbulent* flow of an incompressible fluid. The equation, generally referred to as the Navier–Stokes equation, has been known since Navier (1823):

$$\partial_t \boldsymbol{v} + \boldsymbol{v}\cdot\nabla\boldsymbol{v} = -\nabla p + \nu\nabla^2\boldsymbol{v}, \qquad (1.2)$$

$$\nabla\cdot\boldsymbol{v} = 0. \qquad (1.3)$$

It must be supplemented by initial and boundary conditions (such as the vanishing of $\boldsymbol{v}$ at rigid walls). We shall come back later to the choice of notation.

Quantum version of the
The Nonlinear Schrödinger (NLS) equation
cousin
of
the Korteweg-de Vries Equation

$$i\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial^2 u}{\partial \xi^2} + k|u|^2 u = 0$$

$$\frac{\partial u}{\partial t} + \frac{\partial^3 u}{\partial \xi^3} + ku\frac{\partial u}{\partial \xi} = 0$$

...START WITH STOKES EQUATIONS TO MODEL WAVE DYNAMICS

......

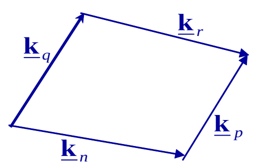
... and by multiple scale perturbation method & MUCH MORE ...

...you get the WAVE TURBULENCE of Zakharov ...

$$\eta(\underline{\mathbf{x}},t)=\frac{1}{\pi}\sum_n\sqrt{\frac{\omega_n}{2g}}\left|B_n\right|\cos\left(\underline{\mathbf{k}}_n\cdot\underline{\mathbf{x}}+\left|\varphi_n\right|\right)+$$

$$+\sum_{n,m}\left|B_n\right|\left|B_m\right|\left[\Gamma_{nm}^{(+)}\cos\left(\underline{\mathbf{k}}_{n+m}\cdot\underline{\mathbf{x}}+\left|\varphi_{n+m}\right|\right)+\Gamma_{nm}^{(-)}\cos\left(\underline{\mathbf{k}}_{n-m}\cdot\underline{\mathbf{x}}+\left|\varphi_{n-m}\right|\right)\right]$$

$$\frac{dB_n}{dt}+i\omega_nB_n=-i\sum_{p,q,r}T_{npqr}\delta_{n+p-q-r}B_p^*B_qB_r$$



Quartet interaction

$$\underline{\mathbf{k}}_n+\underline{\mathbf{k}}_p=\underline{\mathbf{k}}_q+\underline{\mathbf{k}}_r$$

A sea of weakly dispersive nonlinear waves

...moreover for deep water narrowband waves
the Zakharov equation reduces to the NLS equation...

$$i\frac{\partial u}{\partial t}+\frac{1}{2}\frac{\partial^2 u}{\partial \xi^2}+k|u|^2 u=0$$

INTEGRABILITY	NON-INTEGRABILITY
Non-chaos	**Chaos**
Nonlinear interaction of waves and solitons	PERIODIC ORBIT THEORY *Turbulence: walk through a repertoire of recurrent patterns- Cvitanović, GATECH*
NONLINEAR FOURIER ANALYSIS	

Oceanic turbulence of Zakharov → Nonlinear interactions of ‘*stochastic wave groups*’?

Let's start from scratch ….. GAUSSIAN SEAS

$$\eta(\mathbf{x},t)=\sum_{j=1}^{N}a_j\cos\left(\mathbf{k}_j\bullet\mathbf{x}+\omega_j t+\varepsilon_j\right)$$

$$\eta(\mathbf{x},t)$$

Gaussian, ergodic & stationary process of time & space

$$\equiv$$

RANDOM FIELDS

What happens in the neighborhood of a point x_0 if a large crest is recorded in time at x_0 ?

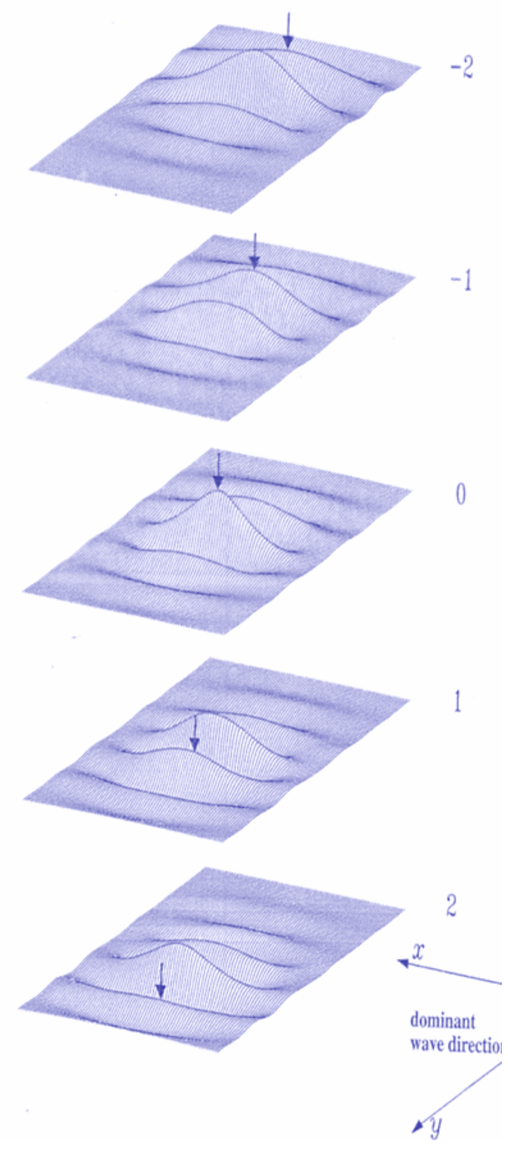
$$\{\eta(\mathbf{x},t) | \eta(\mathbf{x_0},t_0) = h\} = h\,\Psi(\mathbf{x}-\mathbf{x_0}, t-t_0) + \Delta$$

SPACE-TIME covariance

Δ **random residual**, h **Rayleigh variable**

" stochastic wave group "
Boccotti 1989

" Slepian model "
Lindgren 1972, Adler 1981



Generic weakly nonlinear field *

$$\eta = \eta_1 + f(\eta_1) \qquad f(\bullet)\ nonlinear$$

$$O(\varepsilon)\uparrow \qquad O(\varepsilon^2)\uparrow$$

Nonlinear Conditional process

$$\{\eta | \eta(x_0, t_0) = h\} = \{\eta | \eta_1(x_0, t_0) = h_1\}$$

Difficult to solve …

Easy … nonlinear function of a Gaussian stochastic group

Non-Gaussian stochastic group

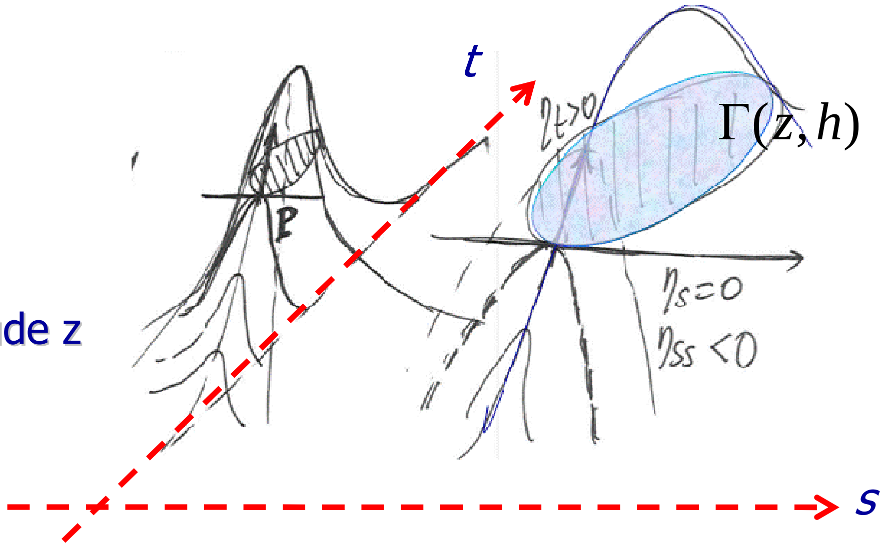
* **Fedele F.** 2008. Rogue waves in oceanic turbulence *Physica D* (in press)

$$S\,\Pr[\eta > h] = \int_h^{\infty} EX_{\max}(z)\,\Gamma(z,h)\,dz$$

stochastic wave group at focusing & Excursion set {η>h}

EXmax(z)=

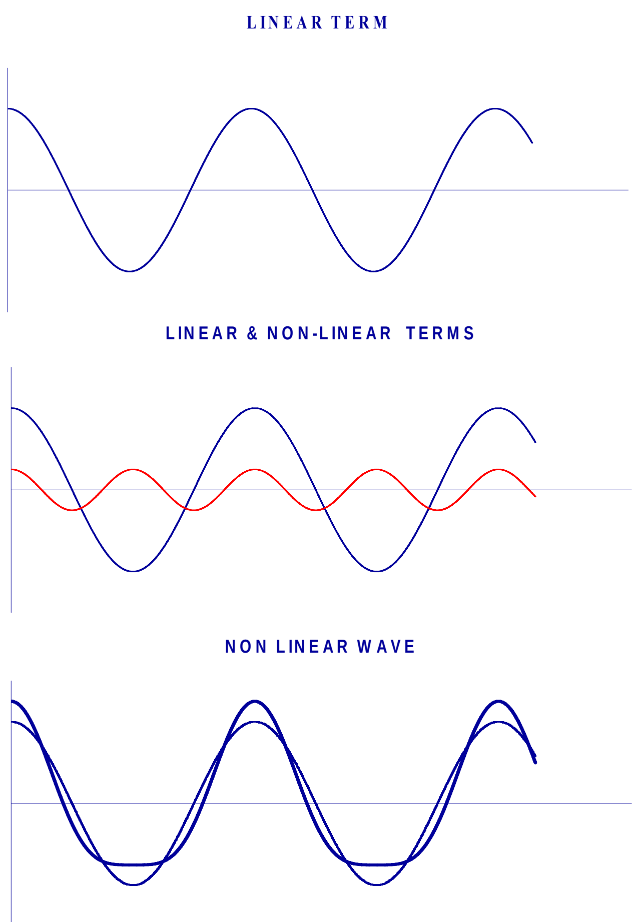
expected number of maxima with amplitude z



$$\Pr[\text{crest} > h] \propto \int_h^{\infty} EX_{\max}(z)dz = \int_h^{\infty} \frac{S}{2\pi}\sqrt{\partial_{tt}F \cdot \partial_{ss}F}\,\frac{d^2\Pr(\eta > z)}{dh^2}dz$$

NONLINEAR EFFECTS TAKEN INTO ACCOUNT

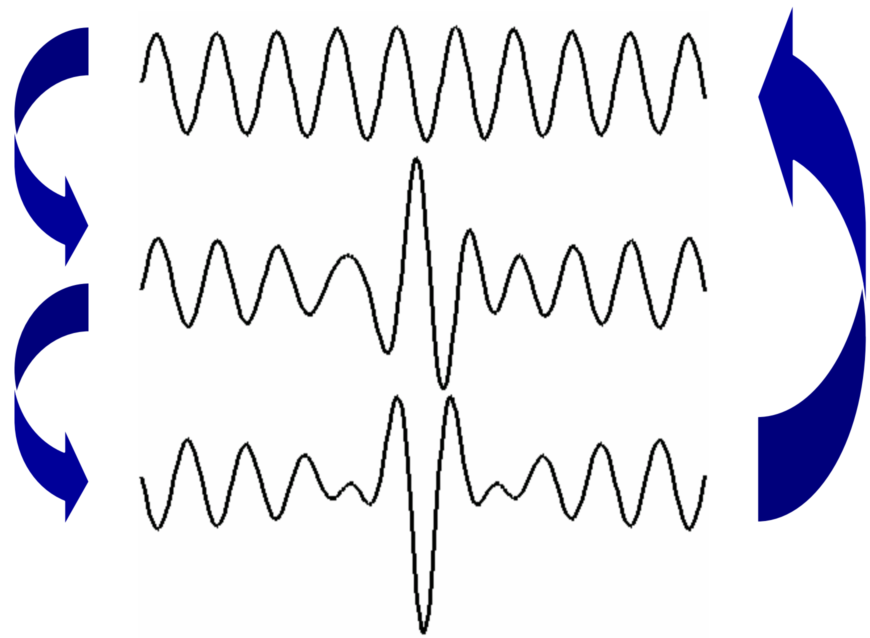
Second order effects: **BOUND WAVES**



Crest–trough asymmetry

skewness>0

Third order effects : **FREE WAVES**



Crest-trough symmetry : kurtosis>3

Benjamin-Feir Index:
BFI=steepness/bandwitdh

DOMINANT ONLY IN
UNIDIRECTIONAL NARROWBAND SEAS !

THE GENERALIZED TAYFUN (GT) DISTRIBUTION*

$$\Pr(crestheight > Z) = \exp\left[-\frac{\left(-1+\sqrt{1+2\mu Z}\right)^2}{2\,\mu^2}\right]\left[1+\frac{\lambda_{40}}{24}Z^2\left(Z^2-3\right)\right]$$

Tayfun distribution

- **GT similar to the GRAM-CHARLIER (GC) approximation of Tayfun & Fedele, OE 2007**

- **GT stems from the Zakharov model with no priori assumptions on the statistical structures of the solution as in GC models**

steepness

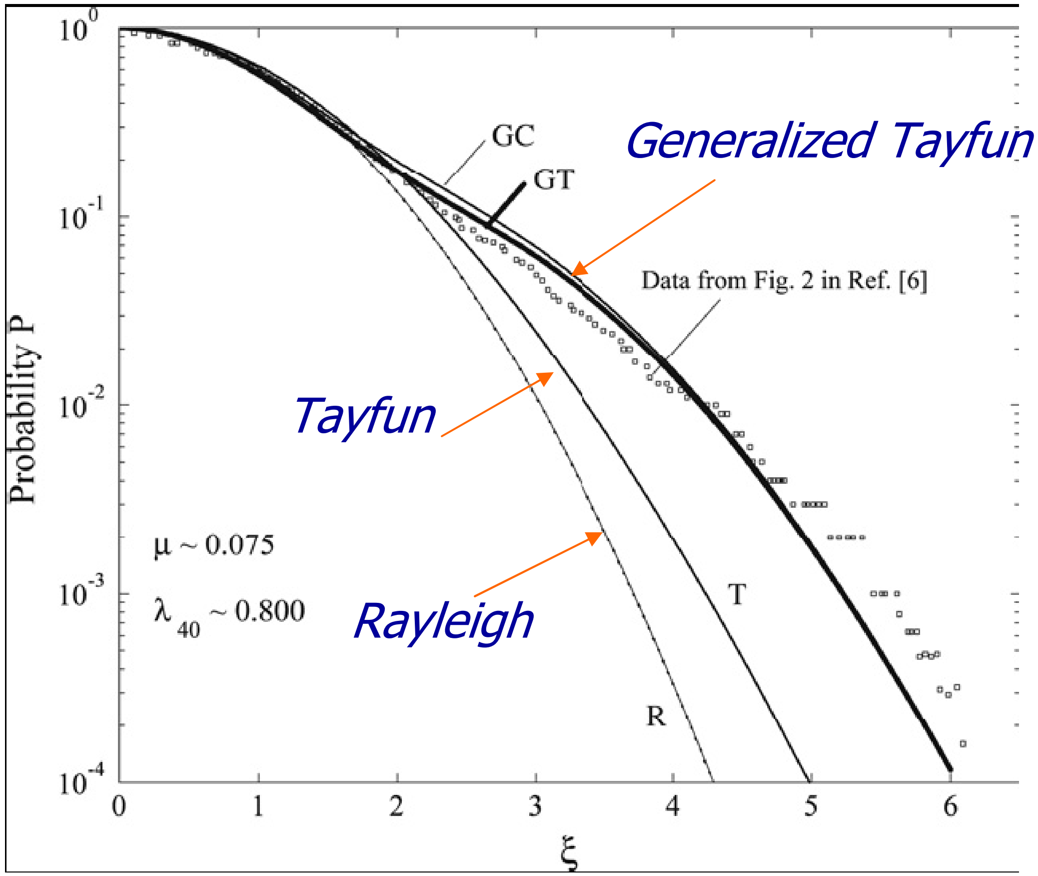
$$\mu \propto \int S_1 S_2 \left[k_1 + k_2 - \left|k_1 - k_2\right|\right] dk_{1,2}$$

kurtosis

$$\lambda_{40} \propto \int K_{12}^{34} S_1 S_2 S_3 \frac{1-\cos\left(\Delta\omega_{1,2}^{3,4} t\right)}{\Delta\omega_{1,2}^{3,4} t}\delta_{12}^{34} dk_{1,2,3}$$

* **Fedele F.** 2008. Rogue waves in oceanic turbulence *Physica D* (in press)

WAVE FLUME DATA COMPARISONS*
(Onorato et al. 2005)



Benjamin-Feir Index
BFI=1.4

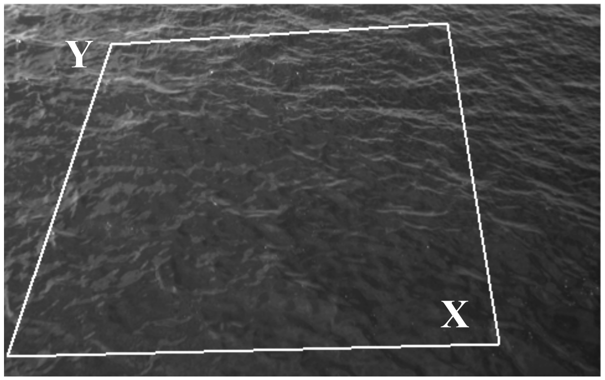
Steepness μ=0.075

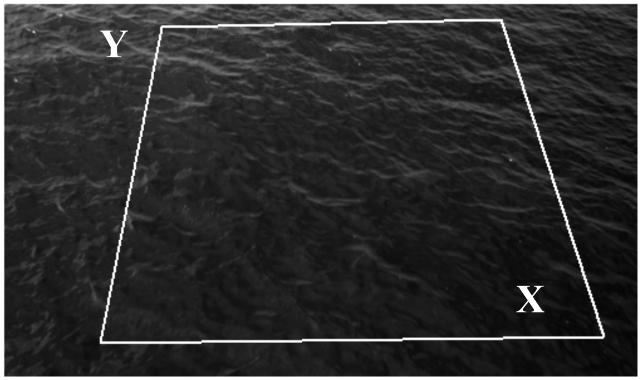
WHAT ABOUT REALISTIC OCEANIC CONDITIONS ?

*** Fedele F.** 2008. Rogue waves in oceanic turbulence *Physica D* (in press)

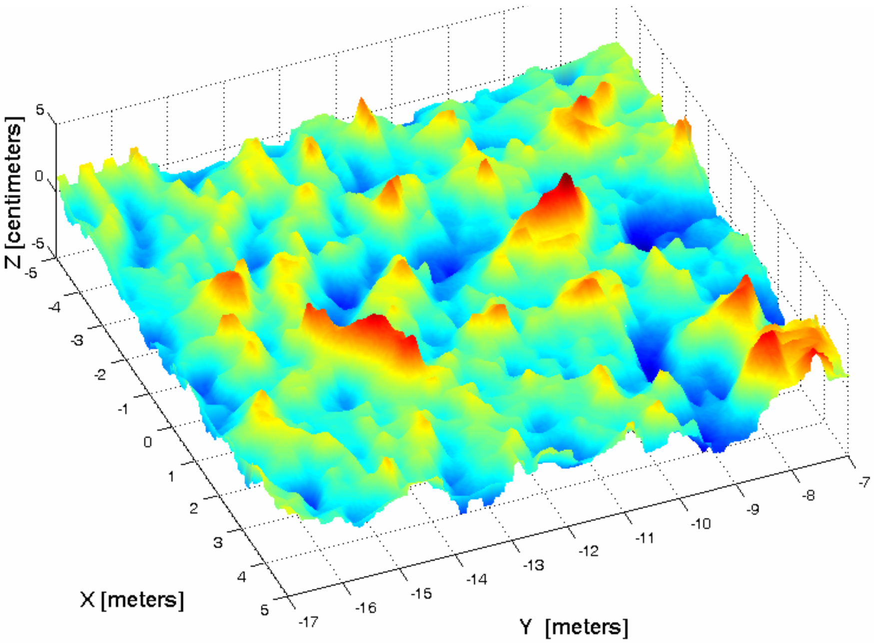
VARIATIONAL WAVE ACQUISITION STEREO SYSTEM (VWASS)

Input stereo pair images. The rectangular domain (8 m x 8.7 m).
The height of the waves is in the range ±0.2 cm.

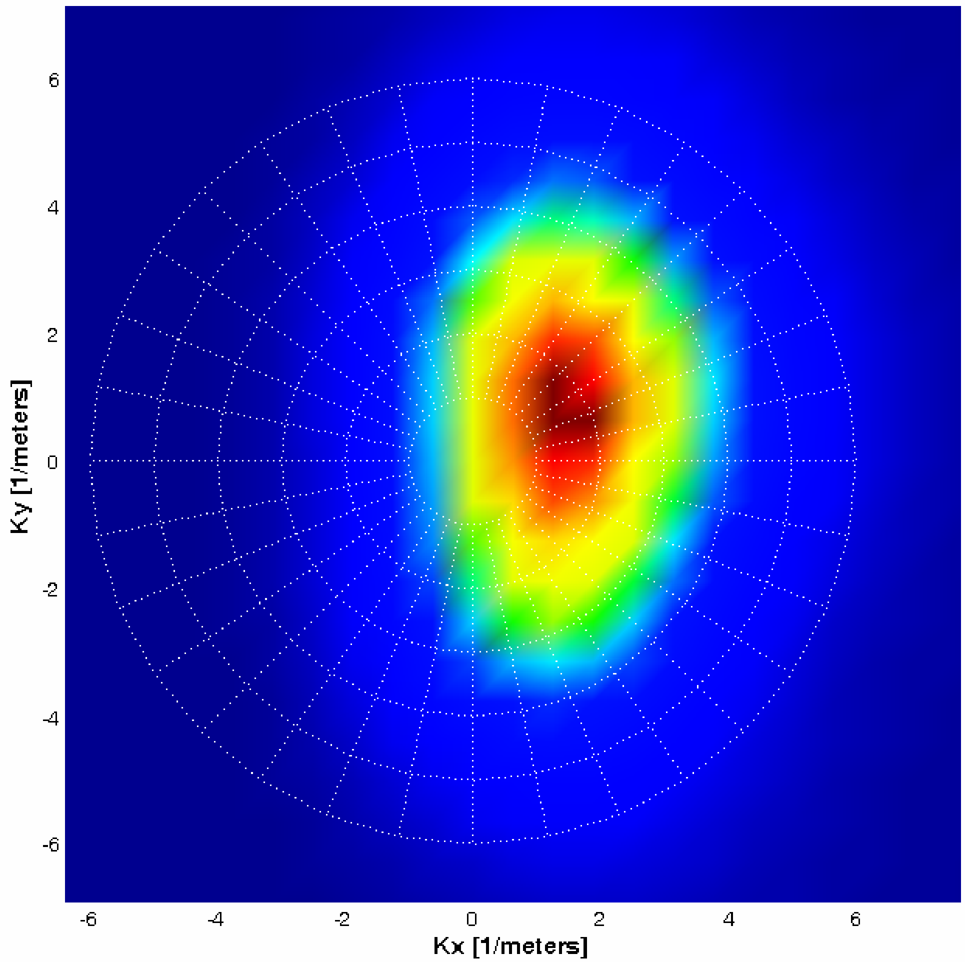


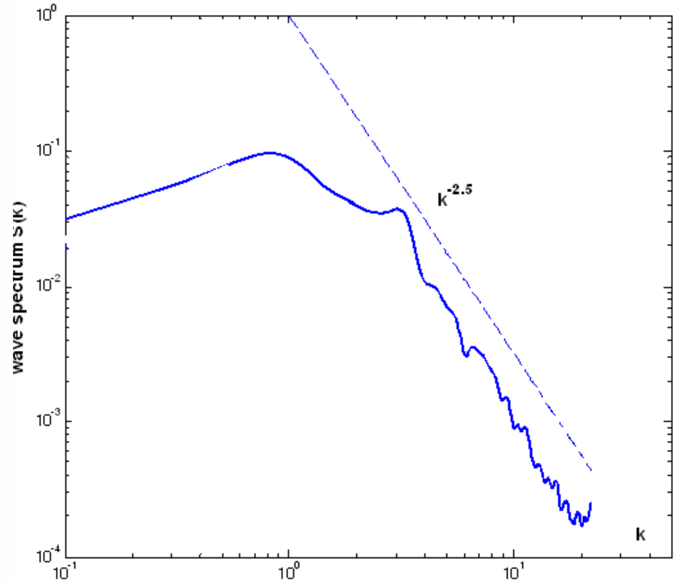


Reconstructed wave surface

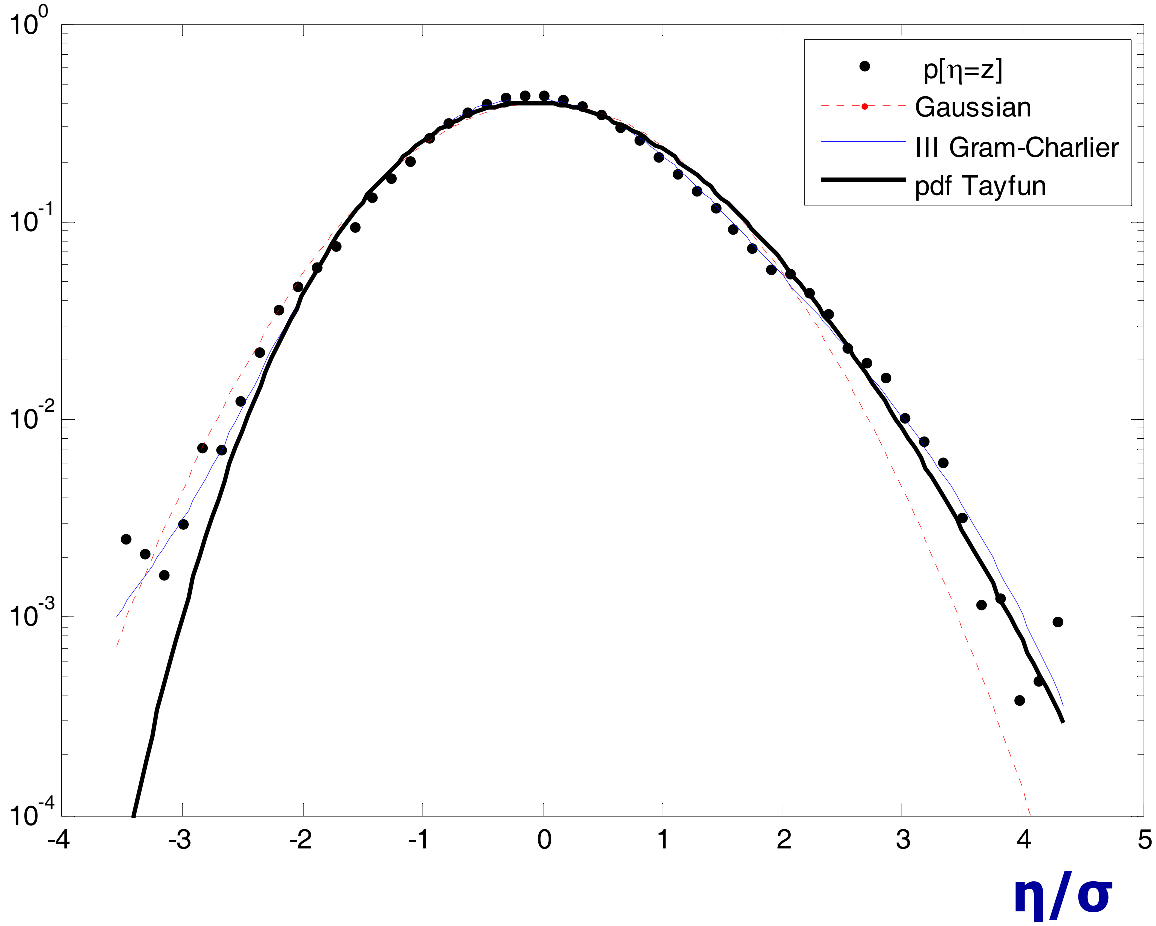


PRELIMINARY RESULTS : wave statistics and spectra





PRELIMINARY RESULTS :Probability density function of wave surface



Generalized Tayfun ~ Tayfun

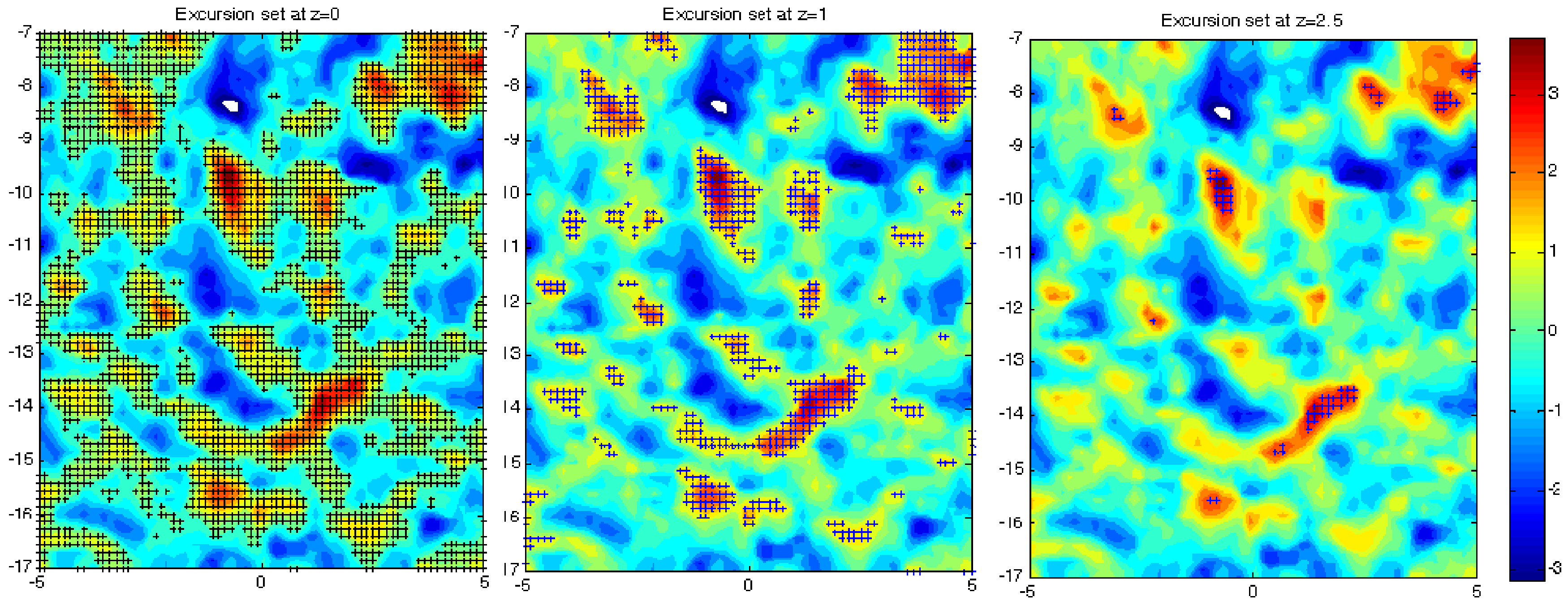
SECOND ORDER EFFECTS DOMINANT !!

BEYOND WAVES & SPECTRA: Euler Characteristic of excursion sets

The geometry of random fields

Adler (1981), Adler, Taylor & Worsley (2007)

EC = #connected components - # holes

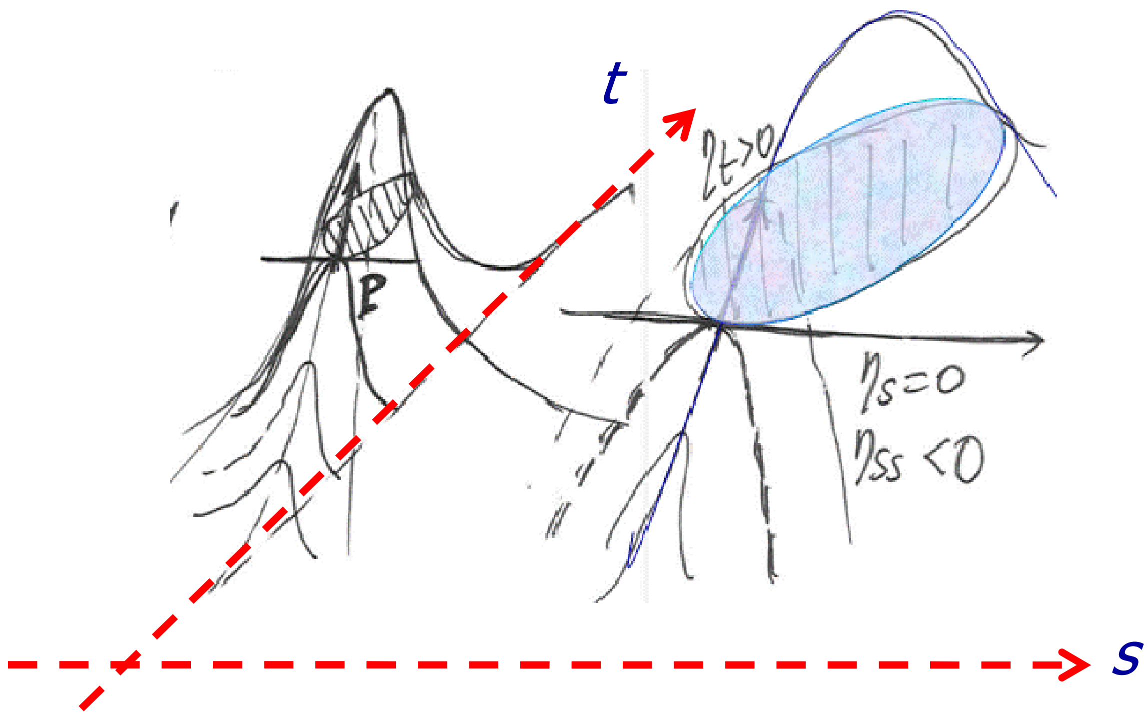


EC counts number of large maxima

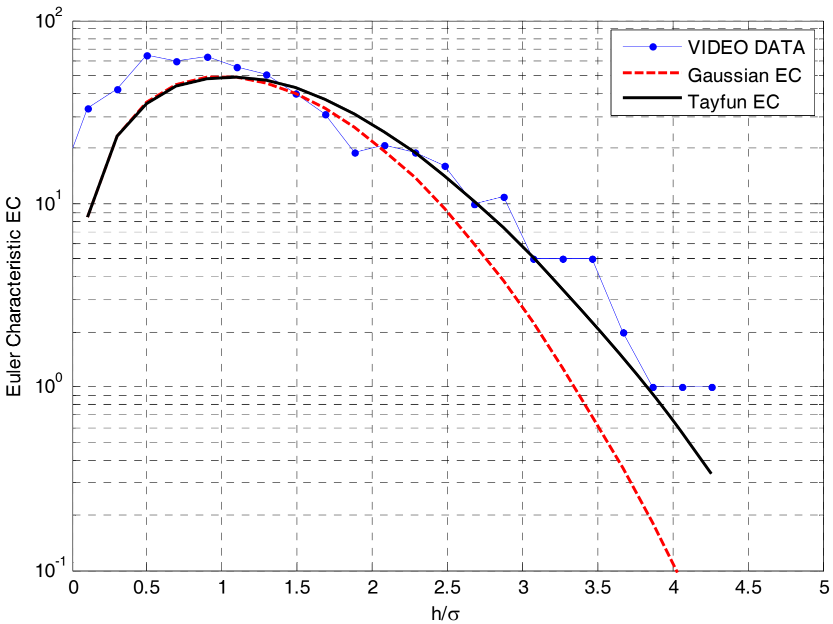
EC counts also large 3D Upcrossings

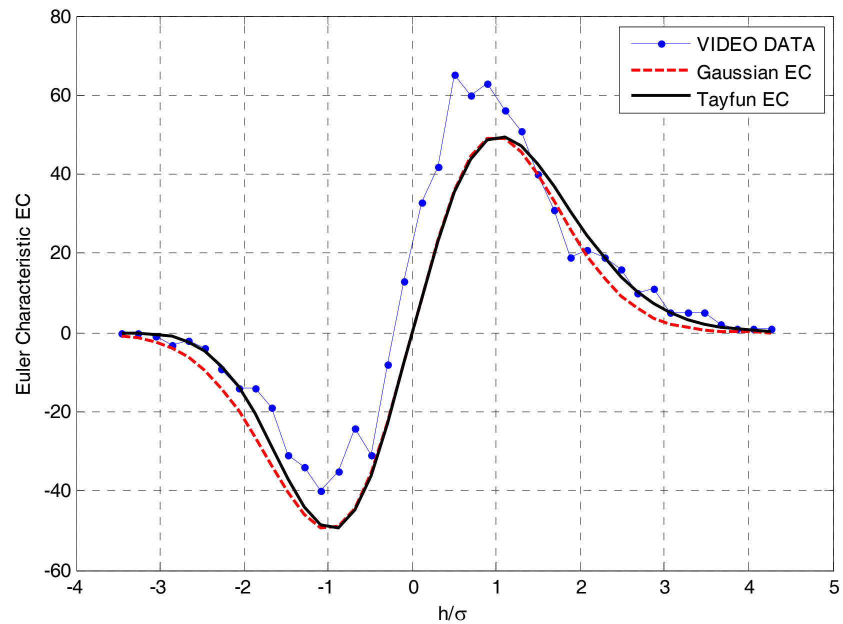
One-to-one correspondence between large maxima & 3D upcrossings

as in one dimensional stochastic processes



Euler characteristic EC of nonlinear wave fields (Piterbarg-Tayfun model)





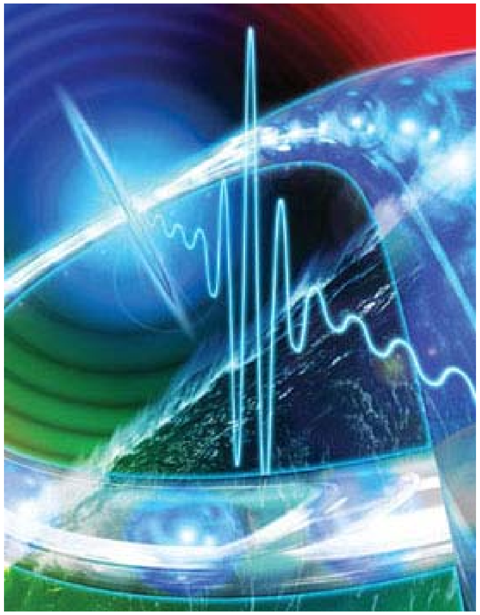
$$EC(h) = Area\left(2\pi\right)^{-3/2}\left|\mathbf{\Lambda}\right|^{1/2}\frac{-1+\sqrt{1+2\mu h}}{\mu}\exp\left[-\frac{\left(-1+\sqrt{1+2\mu h}\right)^2}{2\mu^2}\right]$$

$$\Pr\left[\max_{P\in S}\eta(P) > h\right] \approx EC(h)$$

CONCLUSIONS

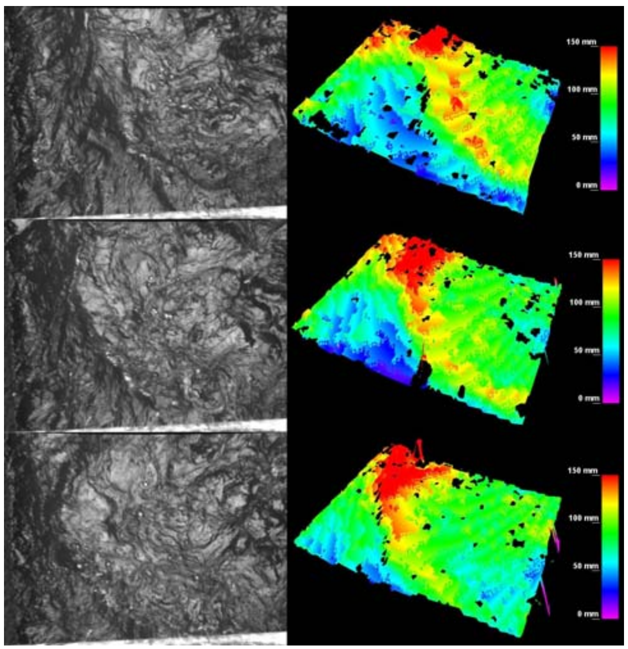
- In open ocean rouge waves appear to be simply rare events of normal populations

- For special wave conditions (undirectional waves in wave flumes) third order resonant interactions are also dominant



ATMOSPHERIC BOUNDARY LAYER & WIND-WAVE INTERACTION

STEREO-VIDEO IMAGERY & HOT-WIRE/SONIC ANEMOMETRY EXPERIMENTS



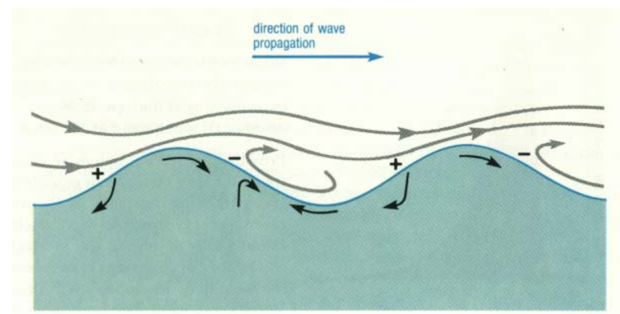


Figure 1.3 Jeffreys' 'sheltering' model of wave generation. Curved lines indicate air flow; short, straight arrows show water movement, which will be explained more fully in Section 1.2.1. The rear face of the wave against which the wind blows experiences a higher pressure than the front face, which is sheltered from the force of the wind. Air eddies are formed in front of each wave, leading to differences in air pressure. The excesses and deficiencies of pressure are shown by plus and minus signs respectively. The pressure difference pushes the wave along.

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ROGUE WAVES , HURRICANE WAVES , GIANT WAVES, FREAK WAVES ....