ROGUE WAVES IN OCEANIC TURBULENCE
&
VORTICES IN AXISYMMETRIC TURBULENT PIPE FLOW:

AN EXTREME VIEW

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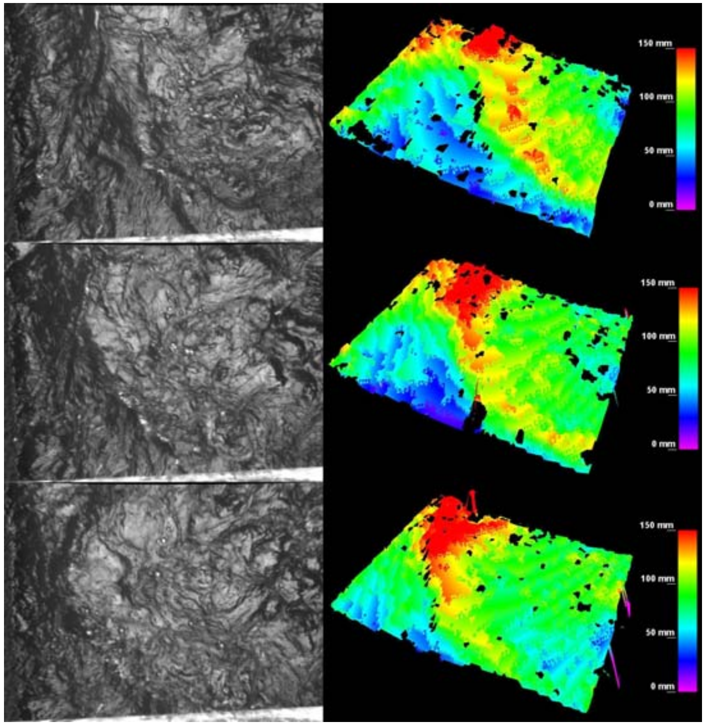
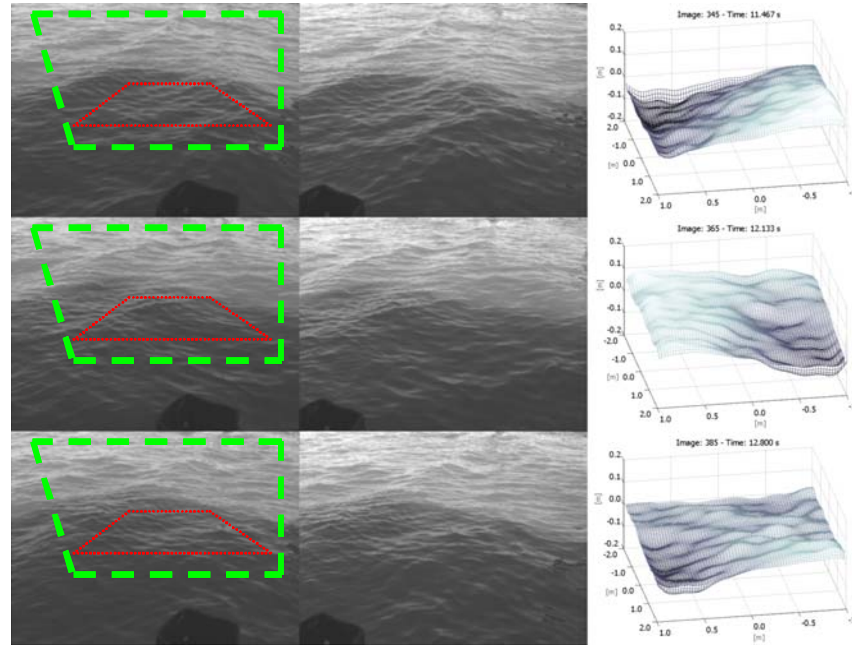
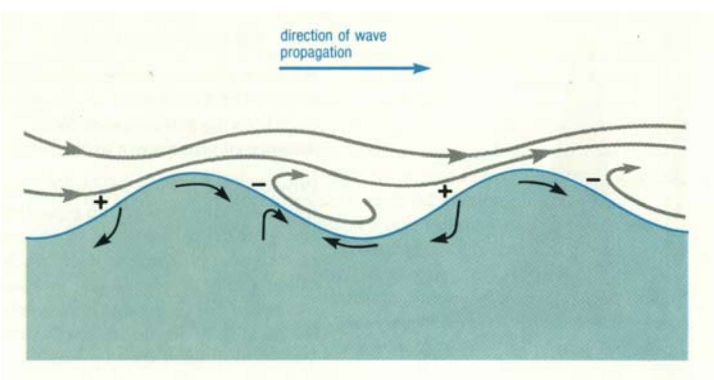
ATMOSPHERIC BOUNDARY LAYER & WIND-WAVE INTERACTION

STEREO-VIDEO IMAGERY & HOT-WIRE ANEMOMETRY EXPERIMENTS





GLOBAL TEAM



F. Fedele GATECH Savannah, Civil Engineering
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A. Tayfun University of Kuwait
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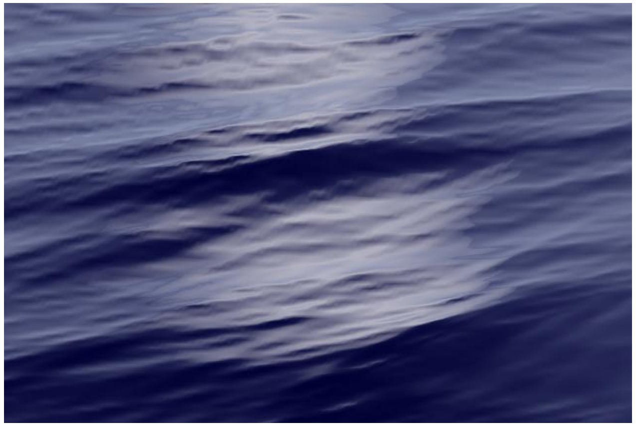
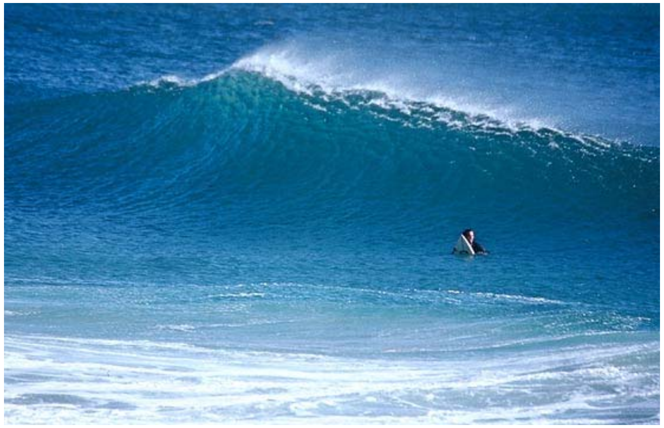
G. Forristall, Forristall Ocean Engineering, Inc. USA

ROGUE WAVES , HURRICANE WAVES , GIANT WAVES, FREAK WAVES ....



A NATURAL BEAUTY !



Freak waves

Rogue waves

Giant waves

Extreme waves

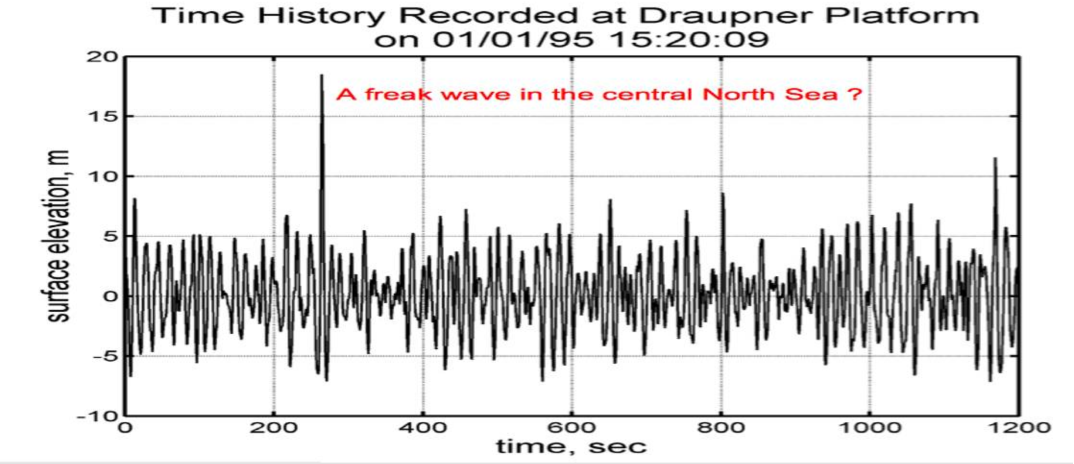
Rogue waves

Extreme waves

Giant waves

Freak waves

DRAUPNER EVENT JANUARY 1995



H_{max}=25.6 m !

Extremely rare event
according to Gaussian model
Probability < 10^{-6} !!!

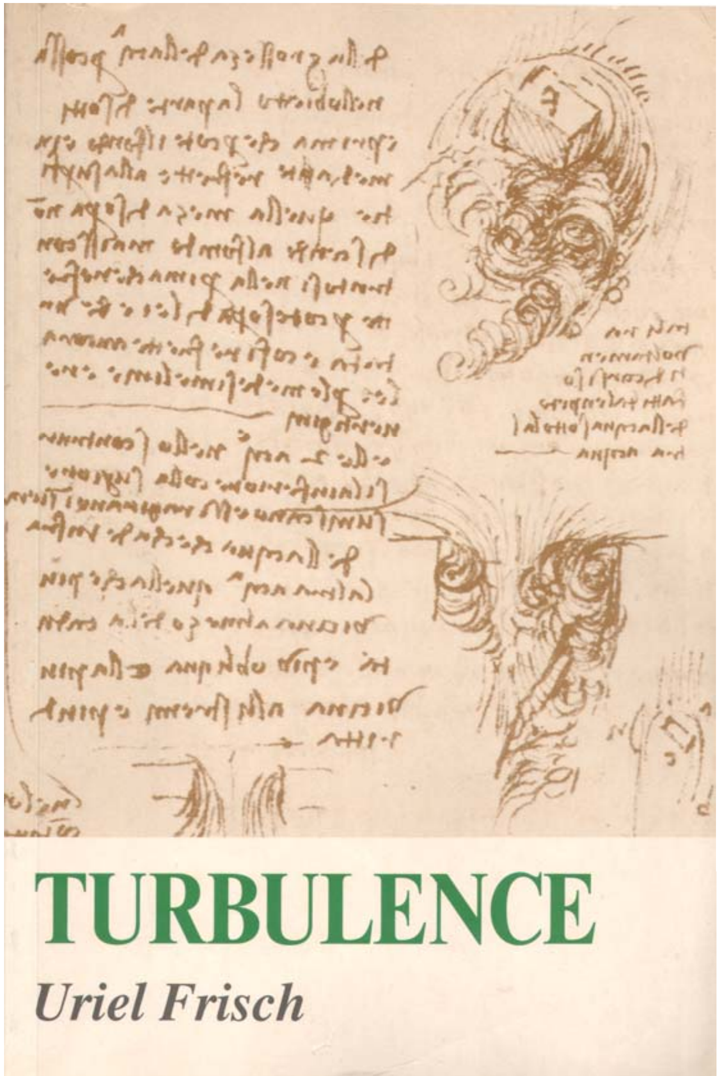
But they still occur in open ocean !

ROGUE WAVES

Rare events of a normal population
or
typical events of a special population ?
(*do we need new physics ?*)

OCEANIC TURBULENCE OF ZAKHAROV
-weak wave turbulence –
- NLS turbulence –

Concept of STOCHASTIC WAVE GROUP
(my contribution)



1.1 Turbulence and symmetries

In Chapter 41 of his *Lectures on Physics*, devoted to hydrodynamics and turbulence, Richard Feynman (1964) observes this:

Often, people in some unjustified fear of physics say you can't write an equation for life. Well, perhaps we can. As a matter of fact, we very possibly already have the equation to a sufficient approximation when we write the equation of quantum mechanics:

$$H\psi = -\frac{\hbar}{i}\frac{\partial \psi}{\partial t}. \tag{1.1}$$

Of course, if we only had this equation, without detailed observation of biological phenomena, we would be unable to reconstruct them. Feynman believes, and this author shares his viewpoint, that an analogous situation prevails in *turbulent* flow of an incompressible fluid. The equation, generally referred to as the Navier–Stokes equation, has been known since Navier (1823):

$$\partial_t \boldsymbol{v} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} = -\nabla p + \nu \nabla^2 \boldsymbol{v}, \tag{1.2}$$

$$\nabla \cdot \boldsymbol{v} = 0. \tag{1.3}$$

It must be supplemented by initial and boundary conditions (such as the vanishing of $\boldsymbol{v}$ at rigid walls). We shall come back later to the choice of notation.

Quantum version of the
The Nonlinear Schrödinger (NLS) equation
cousin
of
the Korteweg-de Vries Equation

$$i\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial^2 u}{\partial \xi^2} + k|u|^2 u = 0$$

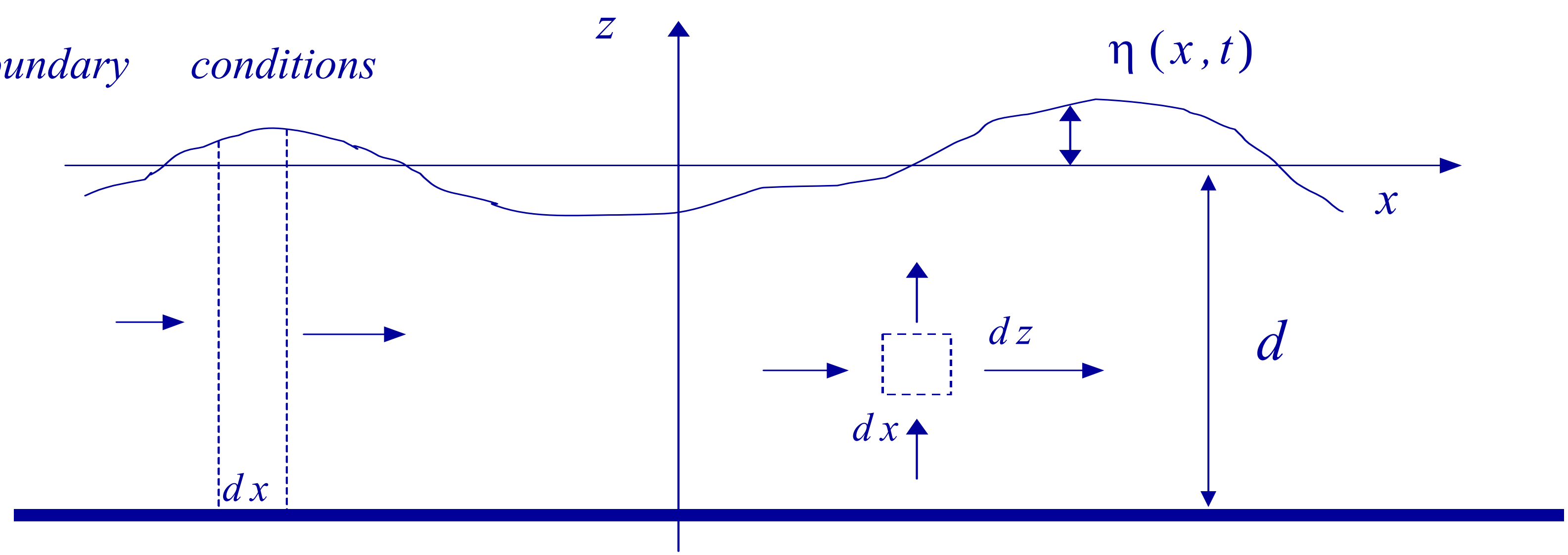
$$\frac{\partial u}{\partial t} + \frac{\partial^3 u}{\partial \xi^3} + ku\frac{\partial u}{\partial \xi} = 0$$

..... START WITH NAVIER-STOKES EQUATIONS TO MODEL WAVE DYNAMICS ......

$$
\begin{cases}
\dfrac{\partial^2 \Phi}{\partial x^2} + \dfrac{\partial^2 \Phi}{\partial z^2} = 0 \\[2ex]
\left(\dfrac{\partial \Phi}{\partial z}\right)_{z=\eta} = \dfrac{\partial \eta}{\partial t} + \dfrac{\partial \eta}{\partial x}\left(\dfrac{\partial \Phi}{\partial x}\right)_{z=\eta} \\[2ex]
\left(\dfrac{\partial \Phi}{\partial t}\right)_{z=\eta} + \dfrac{1}{2}\left[\left(\dfrac{\partial \Phi}{\partial x}\right)^2 + \left(\dfrac{\partial \Phi}{\partial z}\right)^2\right]_{z=\eta} + g\eta = f(t) \\[2ex]
\textit{boundary} \quad \textit{conditions}
\end{cases}
$$

$$v_z = \frac{\partial \Phi}{\partial z} \qquad v_x = \frac{\partial \Phi}{\partial x}$$

Inviscid, irrotational

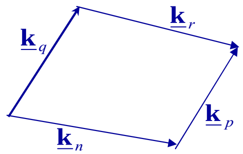


... and by multiple scale perturbation method you get the Zakharov model for WAVE TURBULENCE ....

Third order effects :
FOUR-WAVE RESONANCE
(*WAVE TURBULENCE*)

$$\eta(\underline{\mathbf{x}},t)=\frac{1}{\pi}\sum_n\sqrt{\frac{\omega_n}{2g}}\left|B_n(t)\right|\cos\left(\underline{\mathbf{k}}_n\cdot\underline{\mathbf{x}}+\left|\varphi_n(t)\right|\right)$$

$$\frac{dB_n}{dt}+i\omega_n B_n=-i\sum_{p,q,r}T_{npqr}\delta_{n+p-q-r}B_p^*B_qB_r$$



Quartet interaction

$$\underline{\mathbf{k}}_n+\underline{\mathbf{k}}_p=\underline{\mathbf{k}}_q+\underline{\mathbf{k}}_r$$

Conserved quantities :

Hamiltonian

Wave action

Wave momentum

$$\mathrm{H}=\sum_n\omega_nB_n(t)B_n^*(t)+\frac{1}{2}\sum_{n,p,q,r}T_{npqr}\delta_{n+p-q-r}B_n^*(t)B_p^*(t)B_q(t)B_r(t)$$

$$\mathbf{A}=\sum_nB_n(t)B_n^*(t)\qquad\mathbf{M}=\sum_n\mathbf{k}_nB_n(t)B_n^*(t)$$

Chaotic behavior of a sea of weakly dispersive nonlinear waves

… moreover for narrow-band waves the Zakharov equation reduces to…

$$i\frac{\partial u}{\partial t}+\frac{1}{2}\frac{\partial^2 u}{\partial \xi^2}+k|u|^2u=0$$

In deep water (NLS)

Exact analytical solutions via the Inverse Scattering Transform Technique !

$$\frac{\partial u}{\partial t}+\frac{\partial^3 u}{\partial \xi^3}+ku\frac{\partial u}{\partial \xi}=0$$

In shallow water (KdV)

chaotic behavior due to nonlinear interaction of waves and solitons

↓

NONLINEAR FOURIER ANALYSIS

↓

PERIODIC ORBIT THEORY

Turbulence: walk through a repertoire of recurrent patterns
Cvitanović (GATECH)

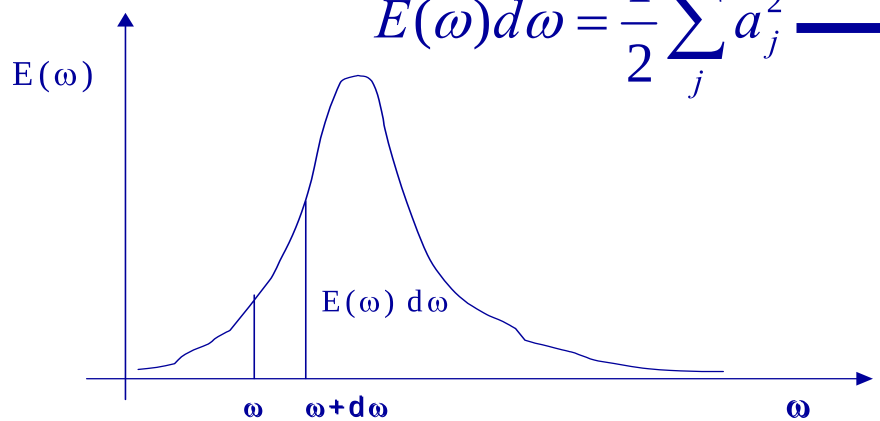
NLS solitons and KdV Cnoidal waves

$$\begin{aligned}
&\operatorname{sech}^2\left(\frac{\sqrt{b_1}\,(x-2tb_1)}{\sqrt{2}}\right)b_1-\\
&\Bigl(2(b_2-b_3)\Bigl(\Bigl(2(b_3-b_1)\Bigl(\operatorname{sech}^2\left(\frac{\sqrt{b_3}\,(x-2tb_3)}{\sqrt{2}}\right)b_3-\operatorname{sech}^2\left(\frac{\sqrt{b_1}\,(x-2tb_1)}{\sqrt{2}}\right)b_1\Bigr)\Bigr)\Bigr)\Big/\\
&\Bigl(\sqrt{2}\sqrt{b_3}\tanh\left(\frac{\sqrt{b_3}\,(x-2tb_3)}{\sqrt{2}}\right)-\sqrt{2}\sqrt{b_1}\tanh\left(\frac{\sqrt{b_1}\,(x-2tb_1)}{\sqrt{2}}\right)\Bigr)^2-\\
&\Bigl(2(b_1-b_2)\Bigl(b_2\operatorname{csch}^2\left(\frac{\sqrt{b_2}\,(x-2tb_2)}{\sqrt{2}}\right)+\operatorname{sech}^2\left(\frac{\sqrt{b_1}\,(x-2tb_1)}{\sqrt{2}}\right)b_1\Bigr)\Bigr)\Big/\\
&\Bigl(\sqrt{2}\sqrt{b_1}\tanh\left(\frac{\sqrt{b_1}\,(x-2tb_1)}{\sqrt{2}}\right)-\sqrt{2}\coth\left(\frac{\sqrt{b_2}\,(x-2tb_2)}{\sqrt{2}}\right)\sqrt{b_2}\Bigr)^2\Bigr)\Bigr)\Big/\\
&\Bigl((2(b_1-b_2))\Big/\Bigl(\sqrt{2}\sqrt{b_1}\tanh\left(\frac{\sqrt{b_1}\,(x-2tb_1)}{\sqrt{2}}\right)-\sqrt{2}\coth\left(\frac{\sqrt{b_2}\,(x-2tb_2)}{\sqrt{2}}\right)\sqrt{b_2}\Bigr)-\\
&(2(b_3-b_1))\Big/\Bigl(\sqrt{2}\sqrt{b_3}\tanh\left(\frac{\sqrt{b_3}\,(x-2tb_3)}{\sqrt{2}}\right)-\sqrt{2}\sqrt{b_1}\tanh\left(\frac{\sqrt{b_1}\,(x-2tb_1)}{\sqrt{2}}\right)\Bigr)\Bigr)\Bigr)^2
\end{aligned}$$

LINEAR WAVES : GAUSSIAN SEAS

$$\eta(x,t)=\sum_{j=1}^{N} a_j \cos\left(k_j x+\omega_j t+\varepsilon_j\right)$$

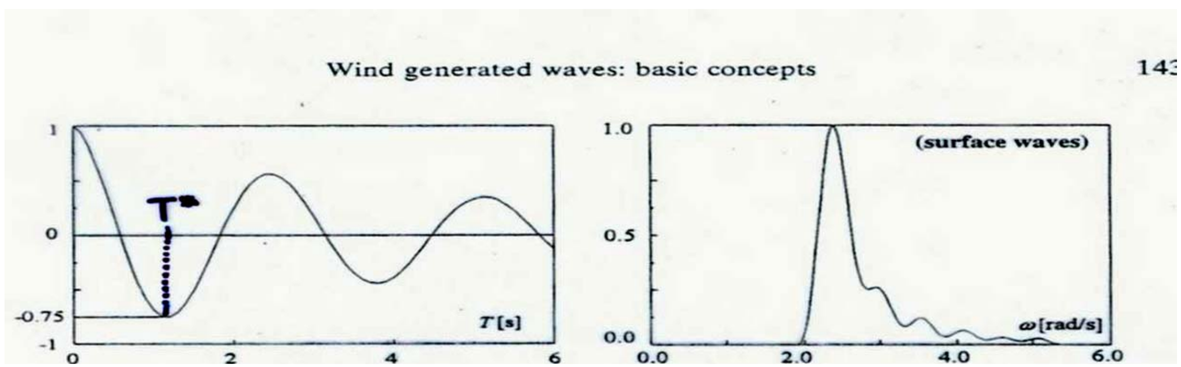
$$E(\omega)d\omega=\frac{1}{2}\sum_j a_j^2 \longrightarrow \psi(T)=\int_0^\infty E(\omega)\cos\omega T\, d\omega$$



$$\eta(x,t)$$

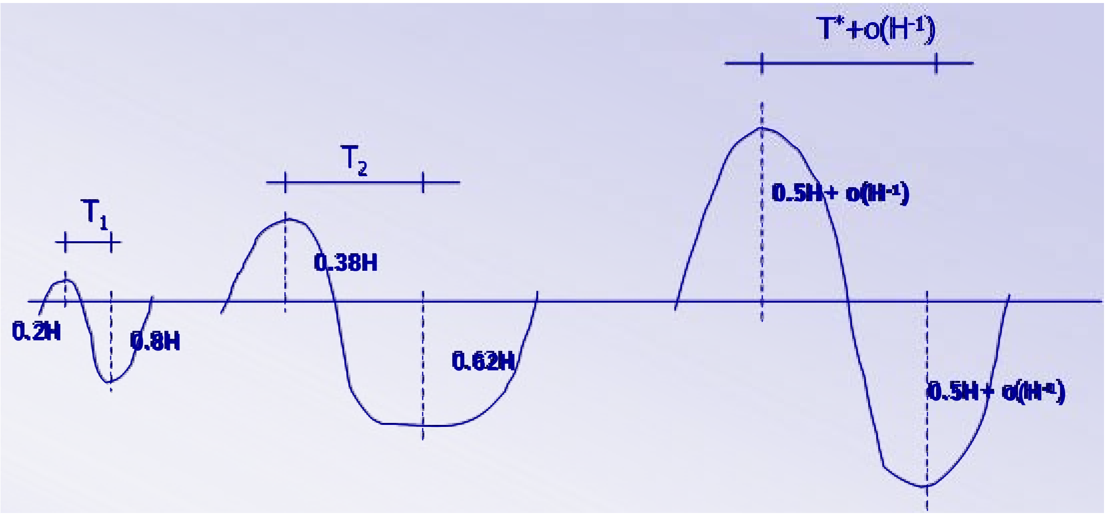
Gaussian, ergodic & stationary process of time

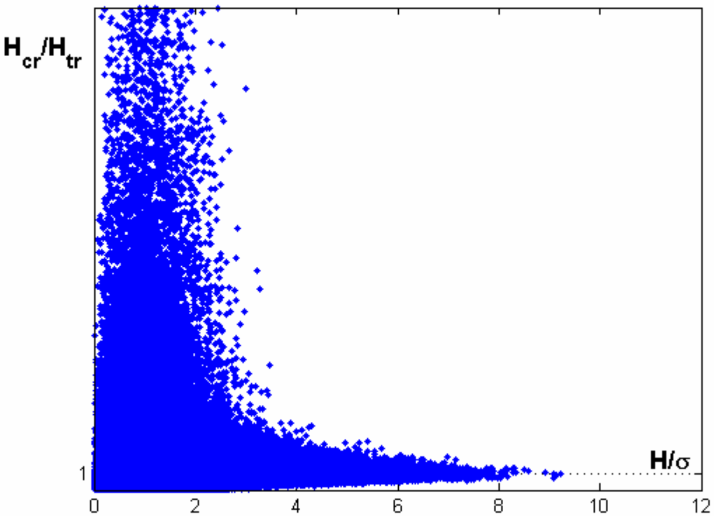
Time covariance

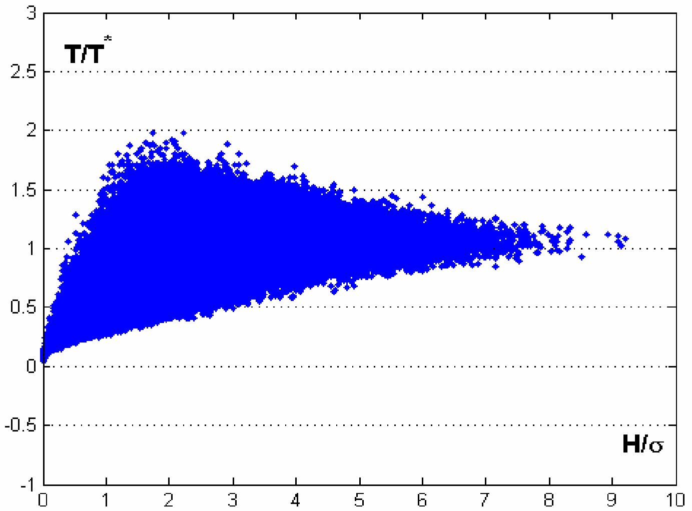


Spectrum

NECESSARY AND SUFFICIENT CONDITIONS FOR THE OCCURRENCE OF A HIGH WAVE IN TIME*







Theory of quasi-determinism, *Boccotti P.* Wave Mechanics *2000 Elsevier

What happens in the neighborhood of a point x_0 if a large crest followed by large trough are recorded in time at x_0 ?

What is the probability that

$$\eta(\mathrm{x}_0 + \mathrm{X}, t_0 + T) \in (u, u + du)$$

conditioned to

$$\eta(\mathrm{x}_0, t_0) = H/2, \;\; \eta(\mathrm{x}_0, t_0 + T_2^*) = -H/2 \;\; ?$$

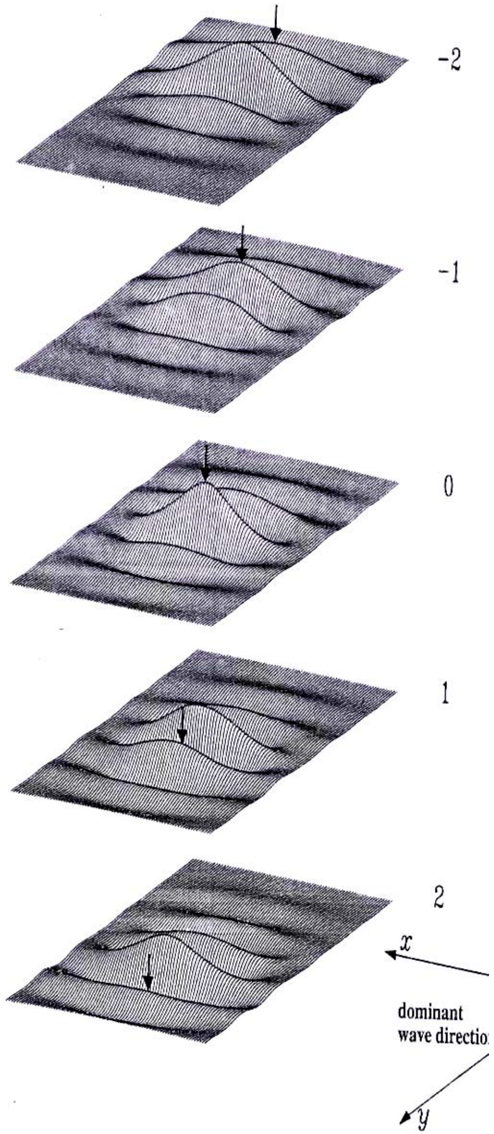
$$h = \frac{H}{\sigma} \to \infty$$

$$\{\eta | \eta(x_0, t_0) = h\} = h\Psi + \Delta$$

Ψ **SPACE-TIME covariance**

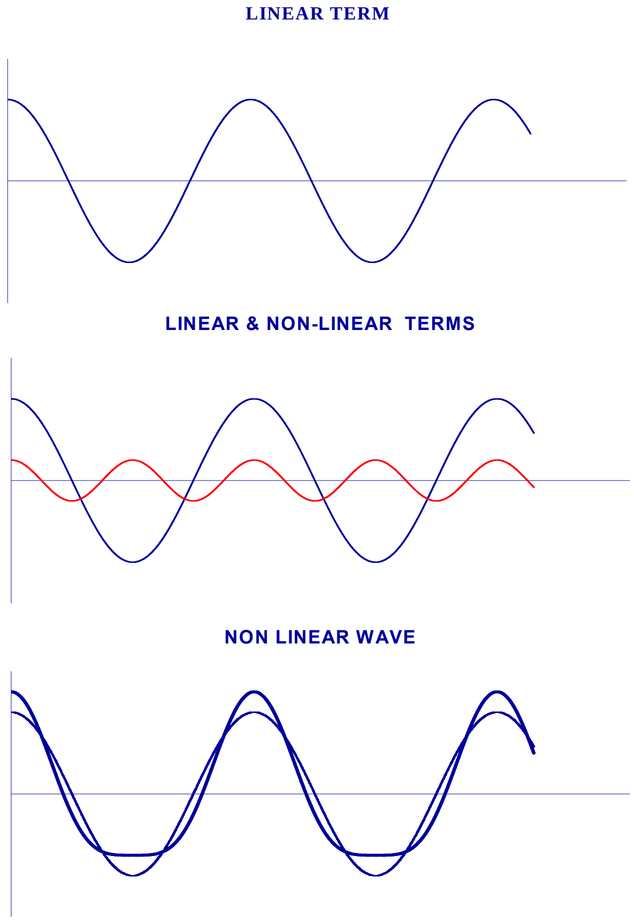
Δ random residual, h Rayleigh variable

stochastic wave group



NONLINEAR RANDOM SEAS

Second order effects: **BOUND WAVES**

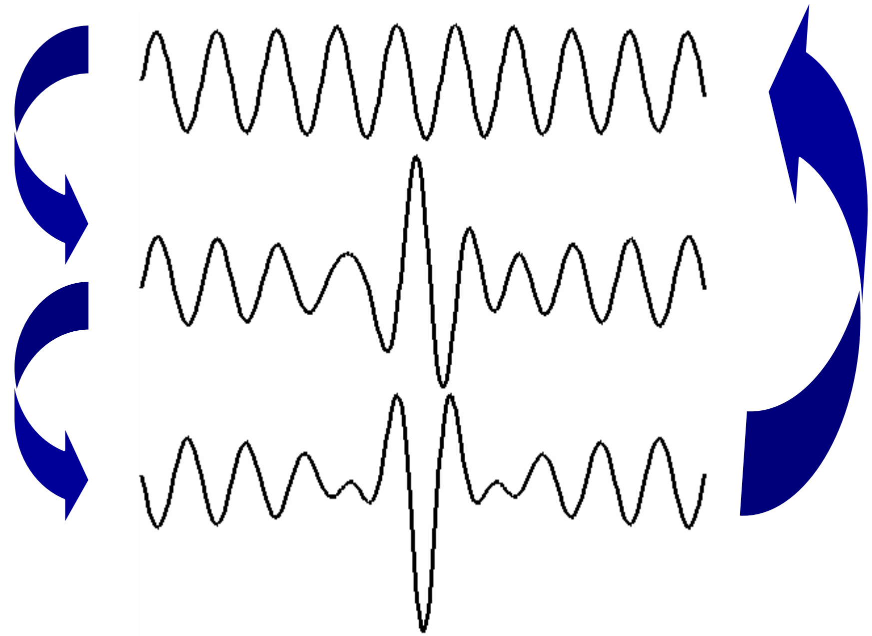


Crest–trough asymmetry : **skewness>0**

Wave height almost linear

<u>Effects on Short time scale</u> : wave period

Third order effects :**FOUR-WAVE RESONANCE**



Crest-trough symmetry : **kurtosis>3**

Benjamin-Feir Index BFI=steepness/bandwitdh

Modulation instability: Fermi Ulam-Pasta recurrence

<u>Effects on slow time scale</u> : wave period/steepness^2

DOMINANT ONLY IN
UNIDIRECTIONAL NARROW-BAND SEAS !

Weak turbulence*

$$\eta = \eta_1 + f(\eta_1) \qquad f(\bullet)\ \textit{nonlinear}$$

$O(\varepsilon)$ ↑ $\quad O(\varepsilon^2)$ ↑

Linear conditional process (Gaussian group)

$$\{\eta_1 | \eta(x_0, t_0) = h_1\} = h_1 \Psi + \Delta$$

Nonlinear Conditional process

$$\{\eta | \eta(x_0, t_0) = h\} = \{\eta | \eta_1(x_0, t_0) = h_1\}$$

↓

Non-Gaussian group $\{\eta | \eta_1(x_0, t_0) = h_1\} = h_1 \Psi + \Delta + f(h_1 \Psi + \Delta)$

*** Fedele F. 2008. Rogue waves in oceanic turbulence Physica D (in press)**

Probability of exceedance for crests:
the generalized Tayfun distribution

$$\Pr(crestheight > Z) = \exp\left[-\frac{1}{2\,\mu^{*2}}\left(-1+\sqrt{1+2\,\mu^{*} Z}\,\right)^{2}\right]\left[1+\frac{\Lambda}{64}\left(Z^{4}-8Z^{2}+8\right)\right]$$

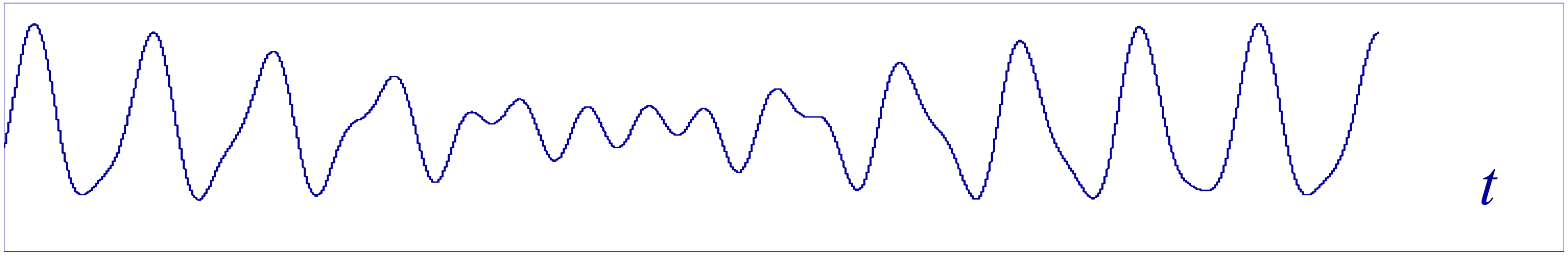
Tayfun distribution

SECOND ORDER EFFECTS + THIRD ORDER EFFECTS

non-resonant interactions — *resonant interactions*

μ=steepness — *BFI=Benjamin-Feir Index*

$$P[Z] = \frac{number\ of\ waves\ with\ crest\ greater\ than\ Z}{total\ number\ of\ waves}$$



WAVE FLUME DATA COMPARISONS*

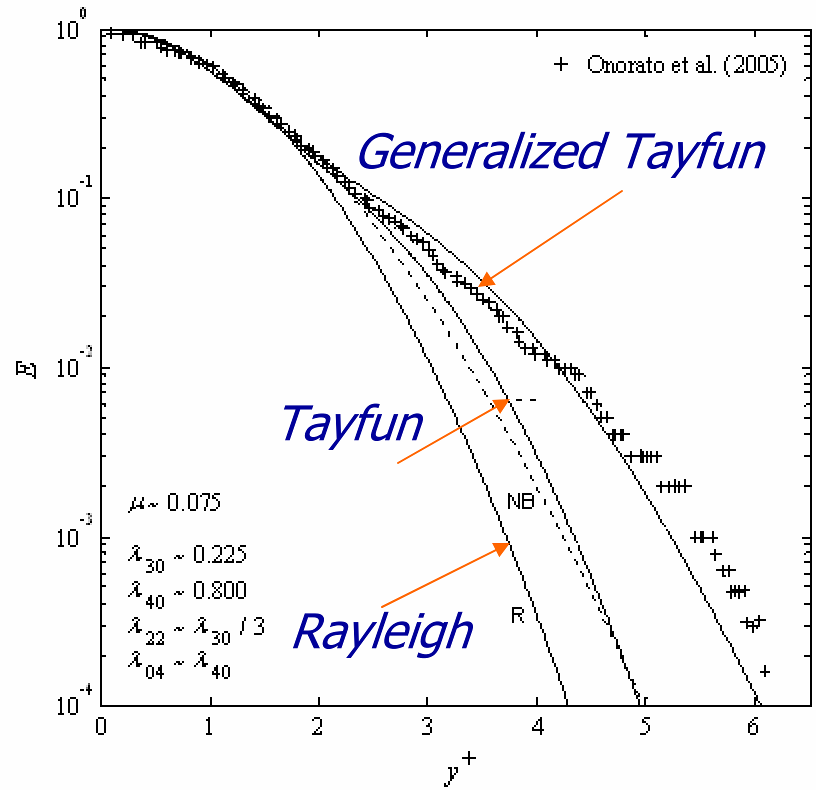
Wave tank experiments:
unidirectional narrow-band seas (Onorato et all. 2005)

unrealistic ocean conditions

THIRD ORDER + SECOND ORDER EFFECTS
BOTH DOMINANT

Benjamin-Feir Index BFI=1.4

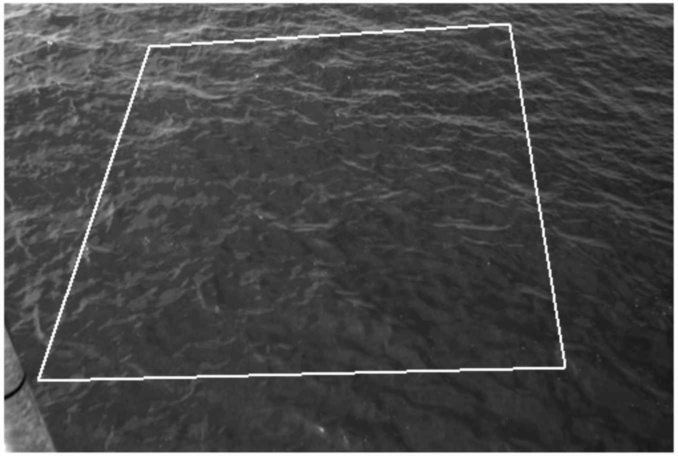
Steepness μ=0.075

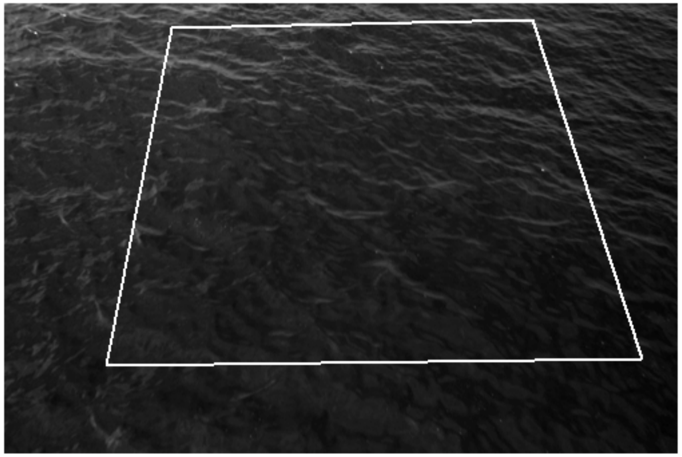


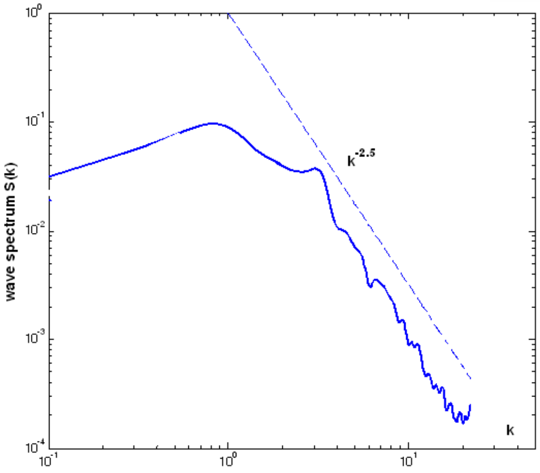
WHAT ABOUT REALISTIC OCEANIC CONDITIONS ?

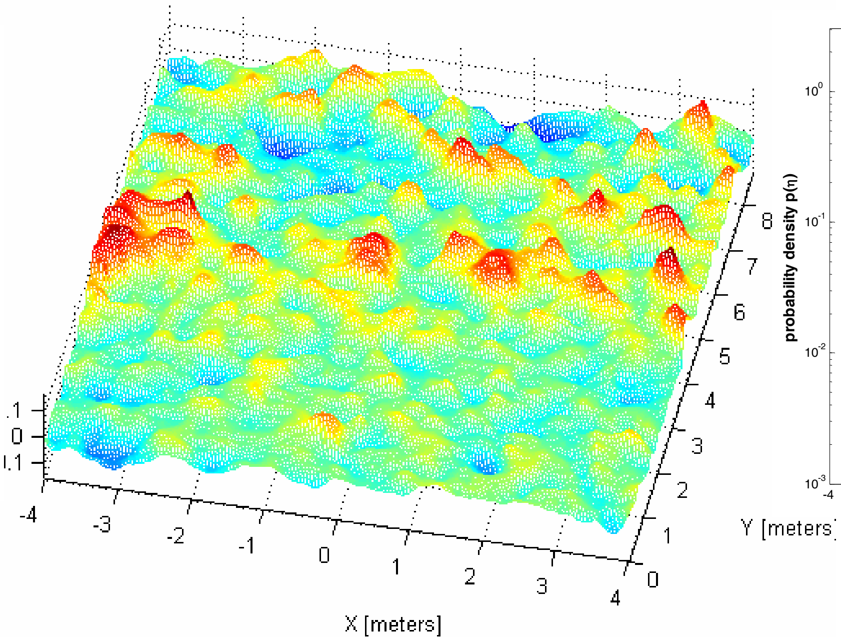
*** Fedele F. 2008. Rogue waves in oceanic turbulence Physica D (in press)**

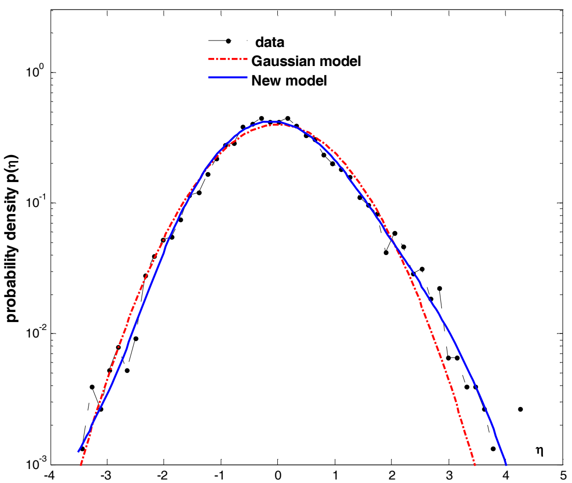
VARIATIONAL WAVE ACQUISITION STEREO SYSTEM



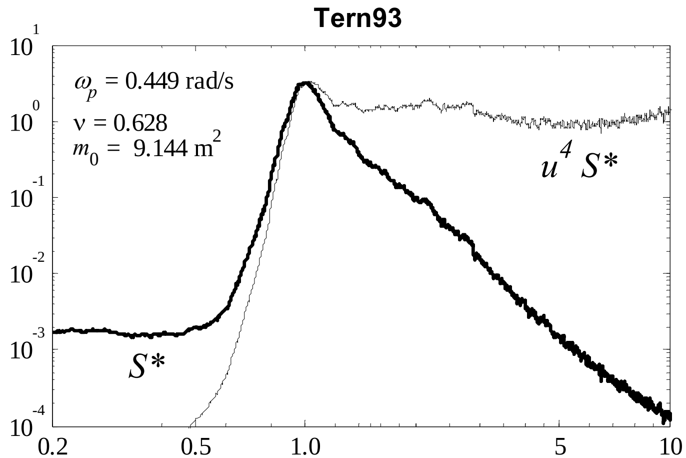


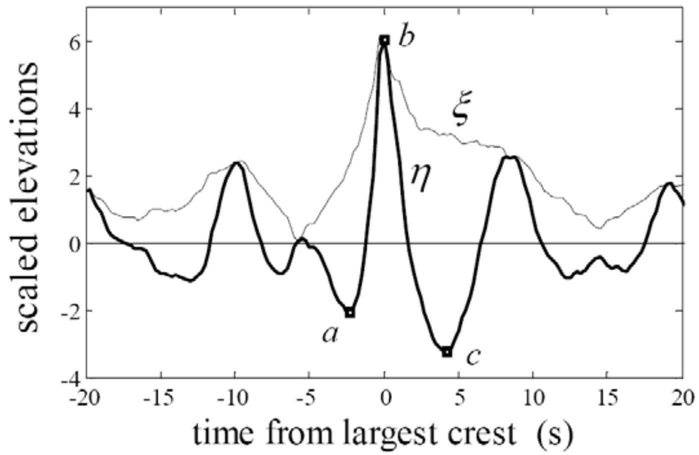


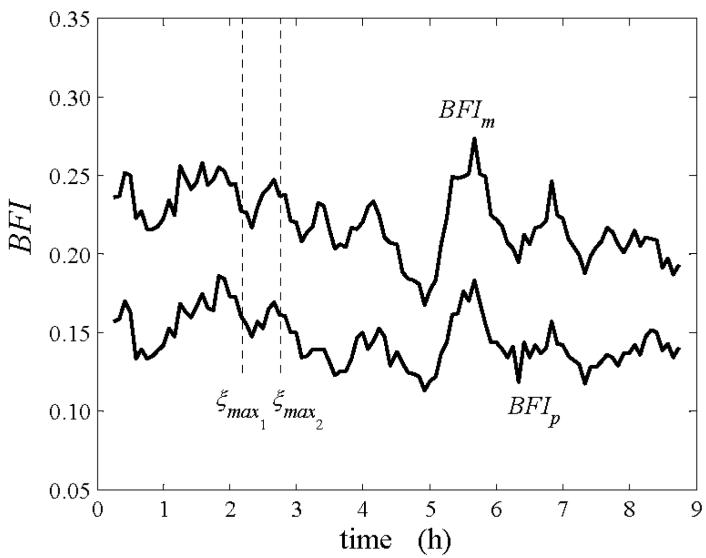


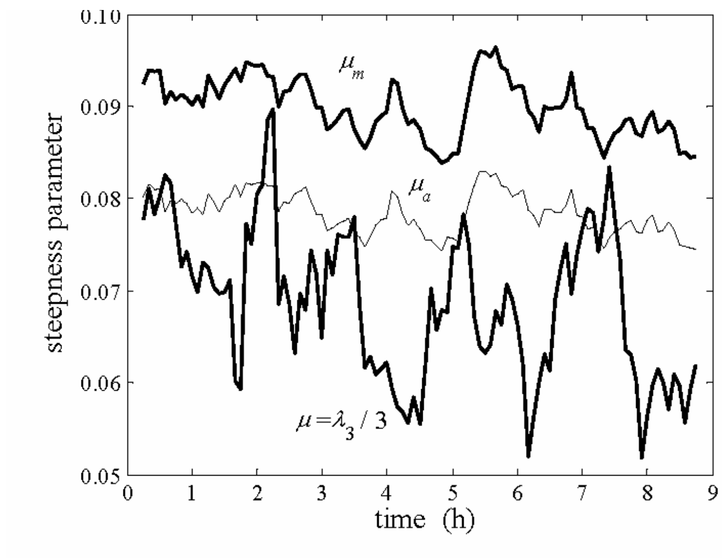


OCEANIC DATA (collected at Tern platform in North Sea)









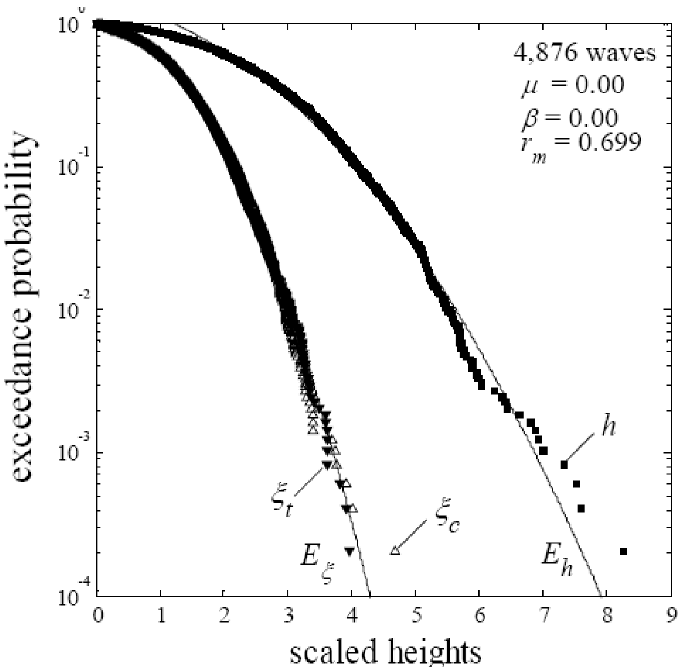
BFI=Benjamin-Feir Index

μ=steepness

resonant interactions

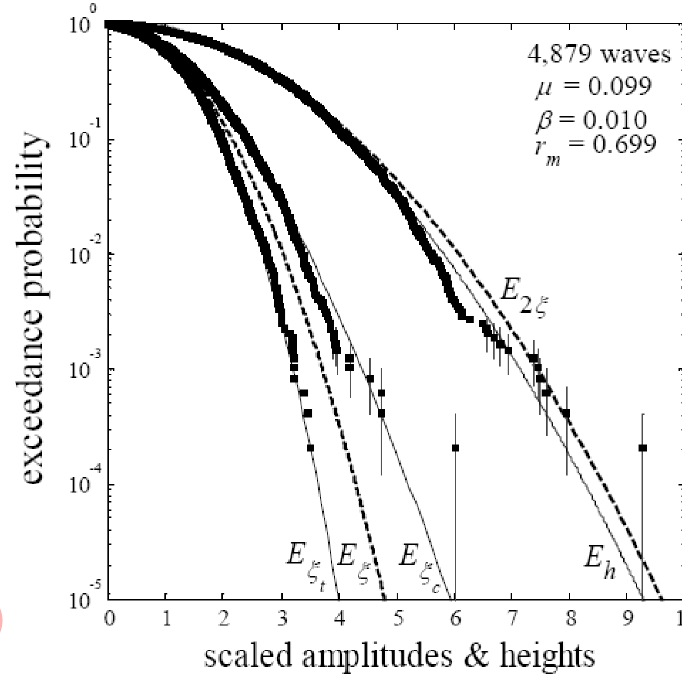
non-resonant interactions

OCEANIC DATA (collected at Tern platform in North Sea)



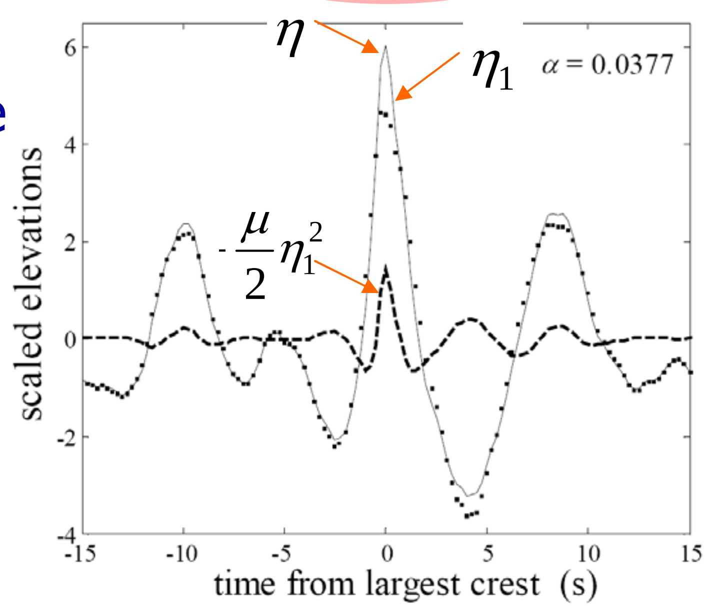
Dominant
II order
non-resonant
Interactions

Generalized Tayfun
&
Tayfun
are the same



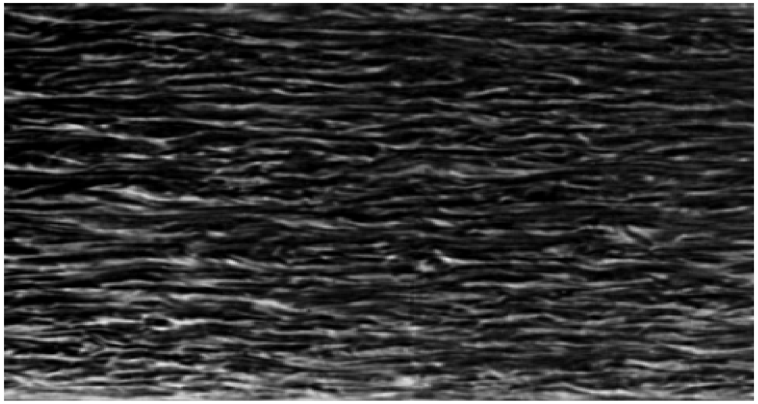
$$\eta \approx \eta_1 + \frac{\mu}{2}\eta_1^2$$

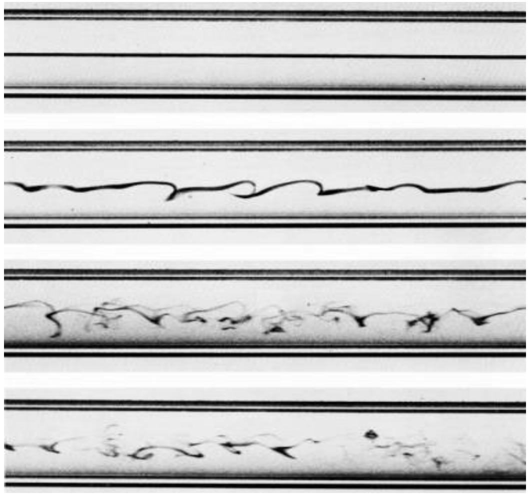
η_1 Linearized Surface Displacement

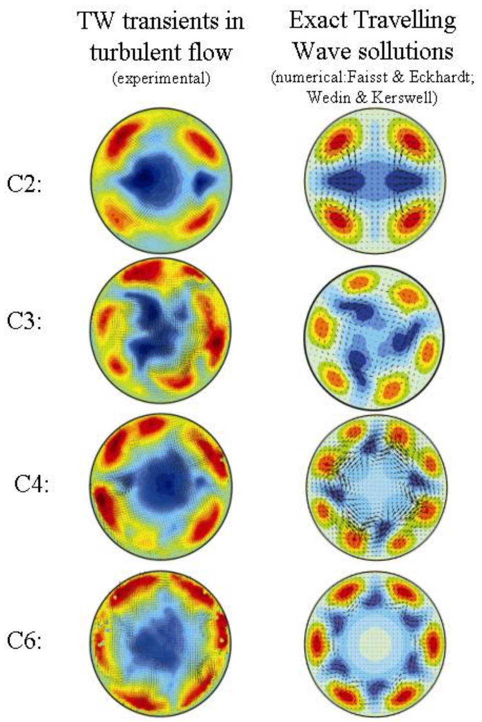


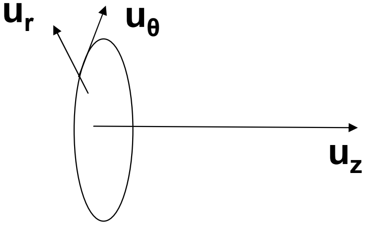
η Nonlinear Surface Dispacement

TURBULENCE IN PIPE FLOWS





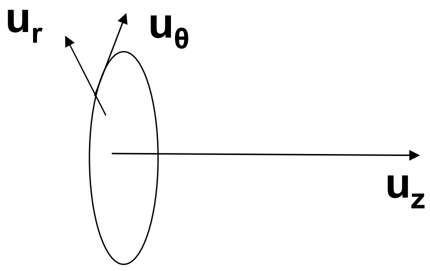




AXISYMMETRIC TURBULENCE or ACADEMIC TURBULENCE

$$G\Psi_t + L\Psi + N\Psi = 0$$

$$G\Psi = \Psi_{rr} - \frac{\Psi_r}{r} - \Psi_{zz}$$



$$L\Psi = W_0\Psi_{zzz} + \left(W_0 G\Psi - \Psi G W_0\right)_z - R_e^{-1} G^2 \Psi$$

$$N\Psi = \Psi_r\left(1/r \cdot G\Psi\right)_z - \Psi_z\left(1/r \cdot G\Psi\right)_r$$

... expand as ....

$$\Psi = \sum A_n(z,t)\psi_n(r)$$

$$\lambda_n G\psi_n + L\psi_n = 0$$

$$A_j(z,t) = \varepsilon^2 a_j\left[\varepsilon\xi, \varepsilon^3 t\right] + ..., \qquad \varepsilon = \mathrm{Re}^{-1} \qquad \mathrm{Re} \to \infty$$

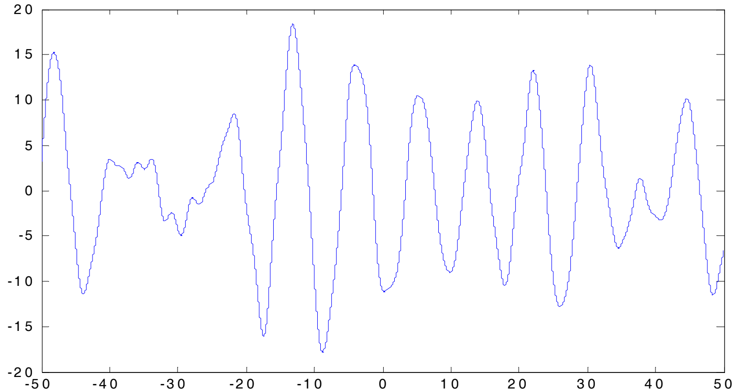
$$\xi = \left(z - c_j t\right) \quad \text{moving frame streamwise direction}$$

KdV system

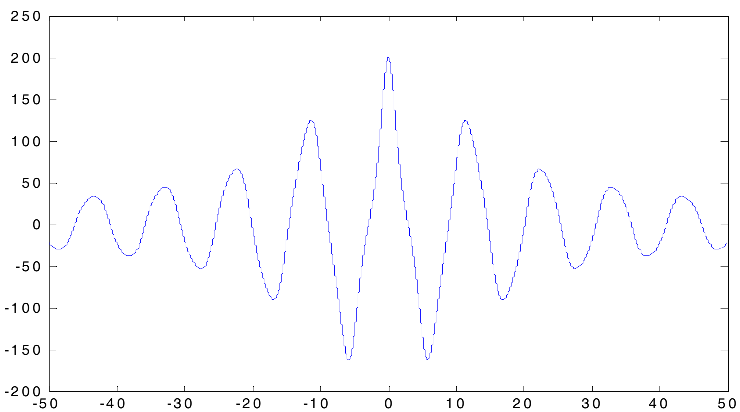
$$\frac{\partial a_j}{\partial t} + \beta_{jj}\frac{\partial^3 a_j}{\partial \xi^3} + \Gamma_{jj} a_j \frac{\partial a_j}{\partial \xi} + \lambda_j a_j = -\sum_m \left(\Gamma_{jm} a_j \frac{\partial a_m}{\partial \xi} + \beta_{jm}\frac{\partial^3 a_m}{\partial \xi^3}\right)$$

chaotic behavior due to nonlinear interactions of *Cnoidal waves*

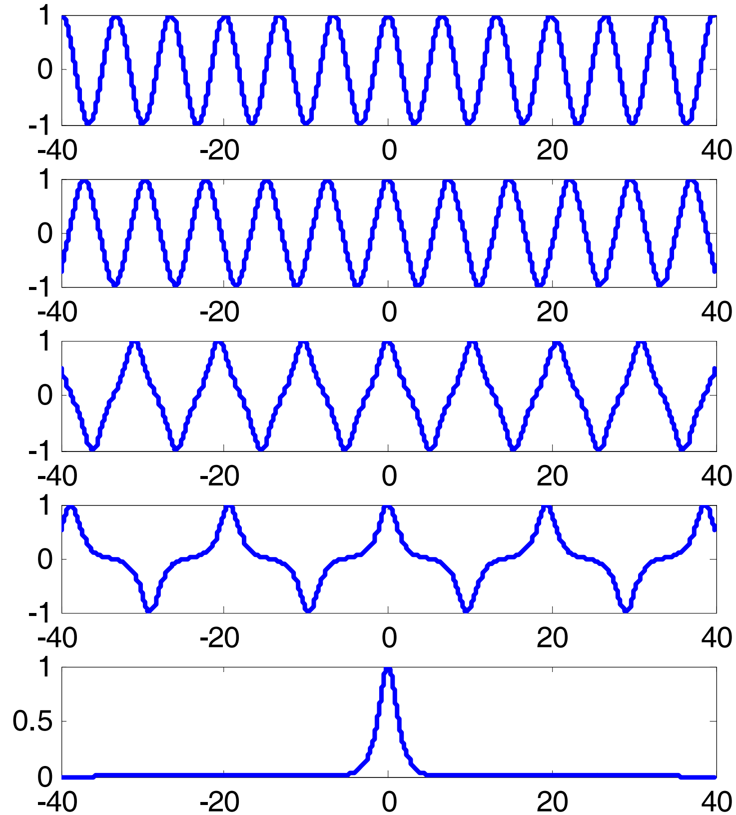
Incoherence



Coherence: Cnoidal wave group



streamwise direction

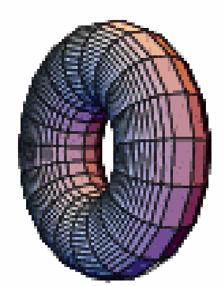


streamwise direction

Fourier waves

Cnoidal waves

Cnoidal solitons



Toroidal vortex tube
Modulated by Cnoidal waves in the streamwise direction

… more work to be done …

CONCLUSIONS:

- In oceanic turbulence second order non-resonant interactions appear dominant

- For special wave conditions (undirectional waves in wave flumes) third order resonant interactions are also dominant

- In open ocean rouge waves appear to be simply rare events of normal populations !

ANY QUESTIONS ?