WAVE GROUPS AND EXTREME EVENTS IN RANDOM SEAS

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Freak waves, rogue waves and giant waves

Nonlinear water waves

Gaussian seas
and extreme waves

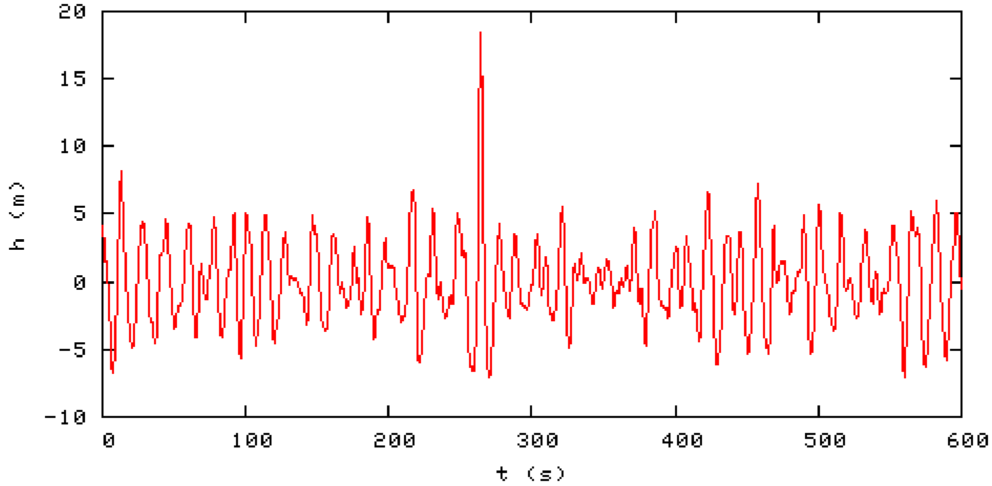
Freak waves

Rogue waves

Giant waves

Extreme waves

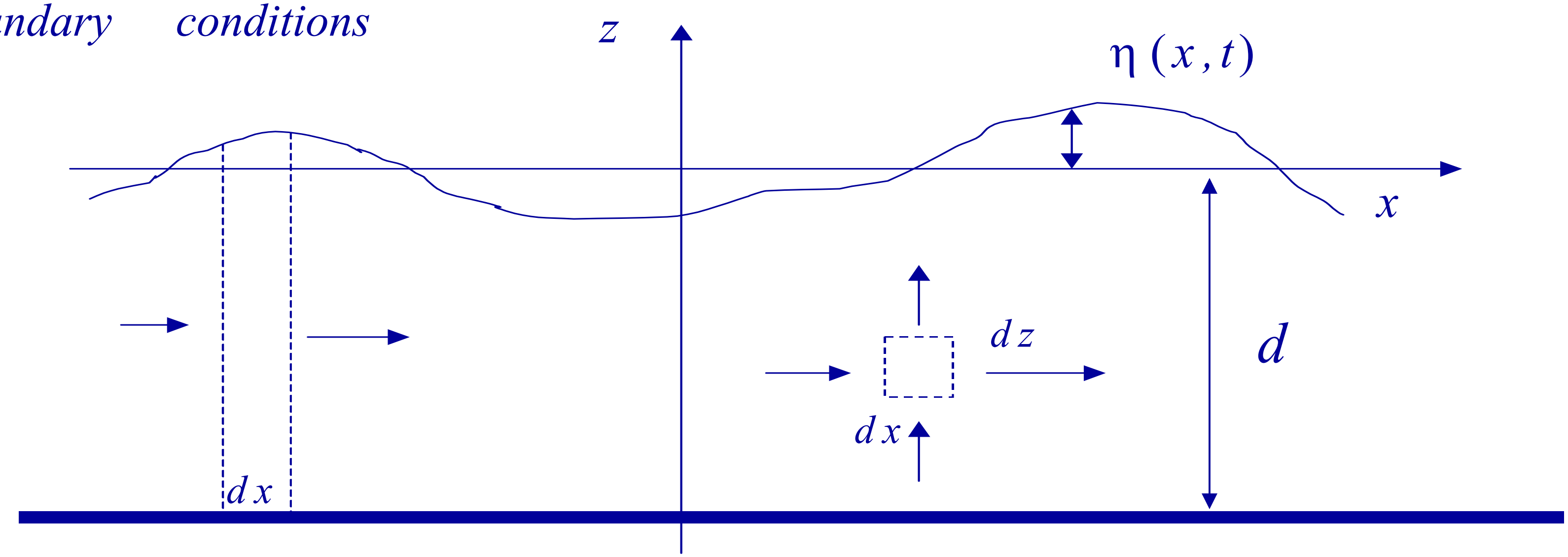
DRAUPNER EVENT JANUARY 1995



STOKES EQUATIONS FOR REGULAR WAVES

$$
\begin{cases}
\dfrac{\partial^2 \Phi}{\partial x^2} + \dfrac{\partial^2 \Phi}{\partial z^2} = 0 \\[2ex]
\left(\dfrac{\partial \Phi}{\partial z} \right)_{z=\eta} = \dfrac{\partial \eta}{\partial t} + \dfrac{\partial \eta}{\partial x} \left(\dfrac{\partial \Phi}{\partial x} \right)_{z=\eta} \\[2ex]
\left(\dfrac{\partial \Phi}{\partial t} \right)_{z=\eta} + \dfrac{1}{2} \left[\left(\dfrac{\partial \Phi}{\partial x} \right)^2 + \left(\dfrac{\partial \Phi}{\partial z} \right)^2 \right]_{z=\eta} + g\eta = f(t) \\[2ex]
\textit{boundary} \quad \textit{conditions}
\end{cases}
$$

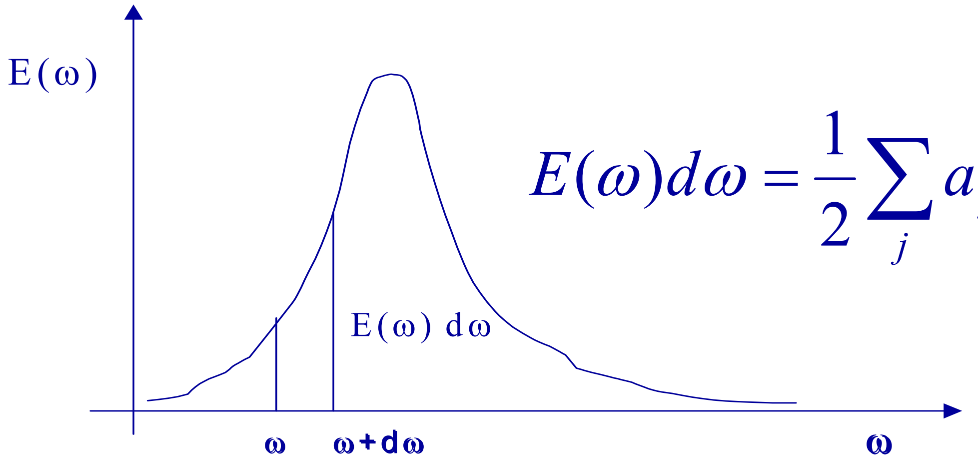
$$v_z = \frac{\partial \Phi}{\partial z} \qquad v_x = \frac{\partial \Phi}{\partial x}$$



GAUSSIAN SEAS

$$\eta(x,t) = \sum_{j=1}^{N} a_j \cos\left(k_j x + \omega_j t + \varepsilon_j\right)$$

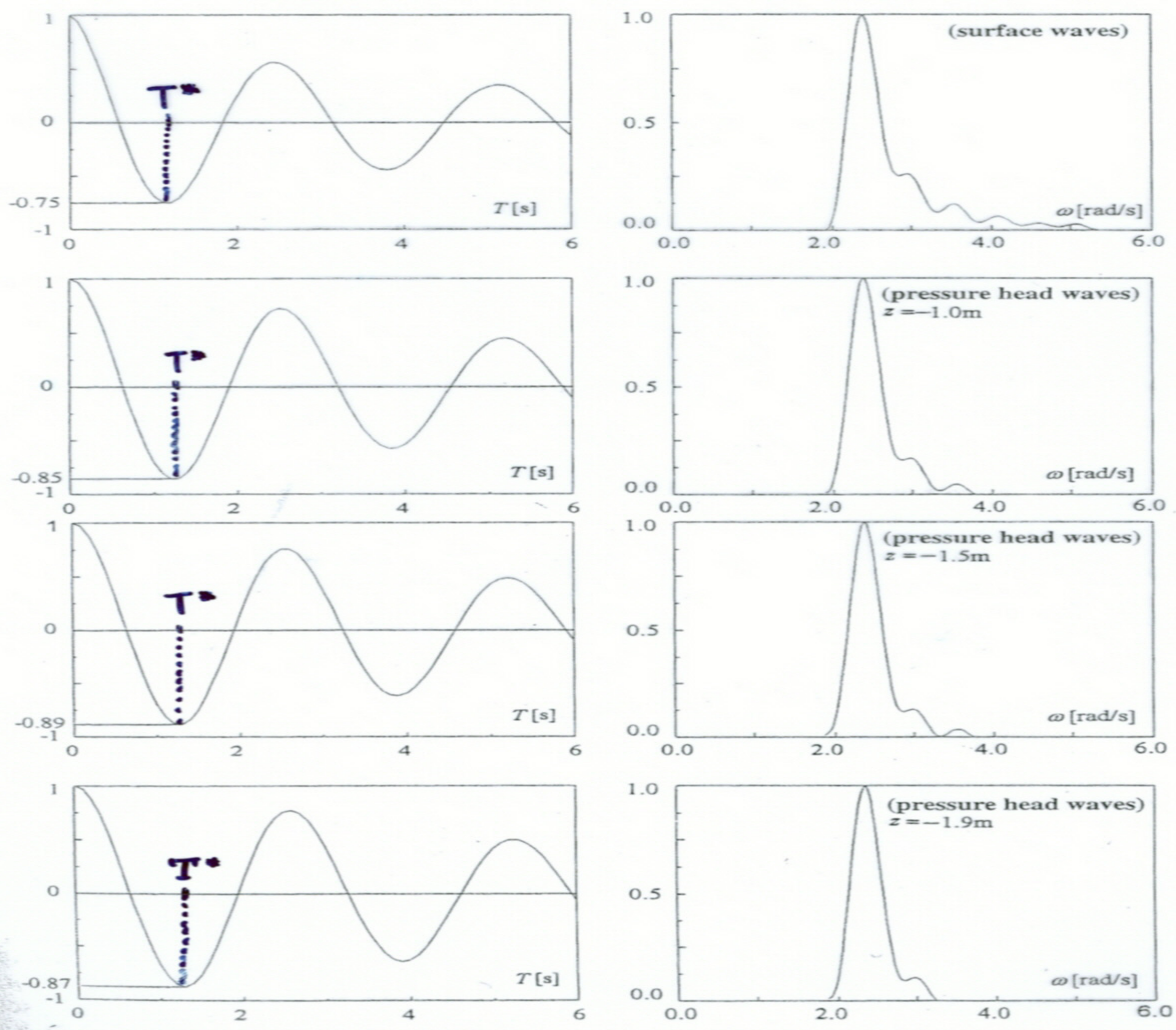
ε_j *uniformly random in* $[0,2\pi]$



$$E(\omega)d\omega = \frac{1}{2}\sum_j a_j^2 \qquad \omega < \omega_j < \omega + d\omega$$

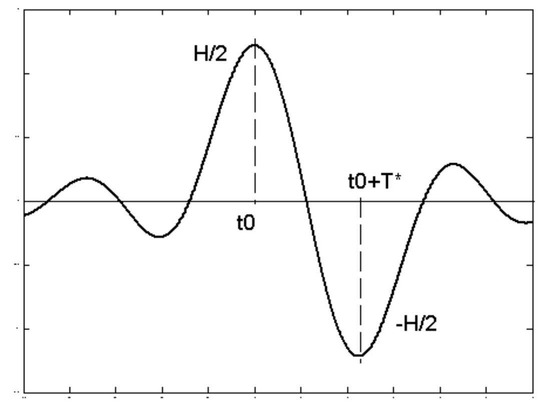
Stationarity Ergodicity Gaussianity

TYPICAL WAVE SPECTRA FROM MEDITERRANEAN SEA

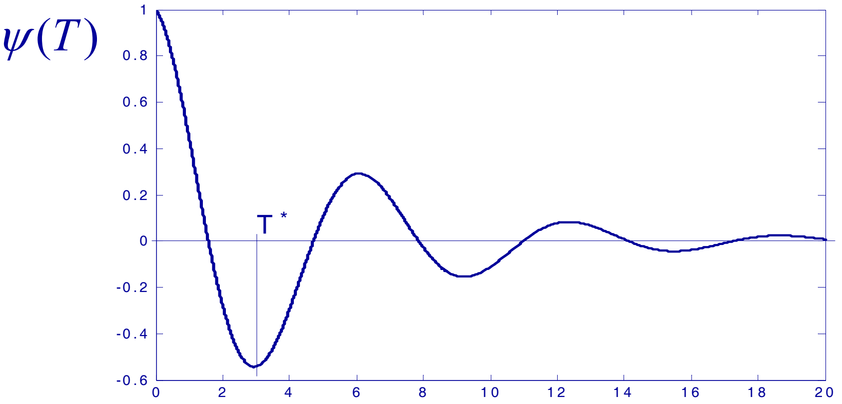


TIME DOMAIN ANALYSIS :

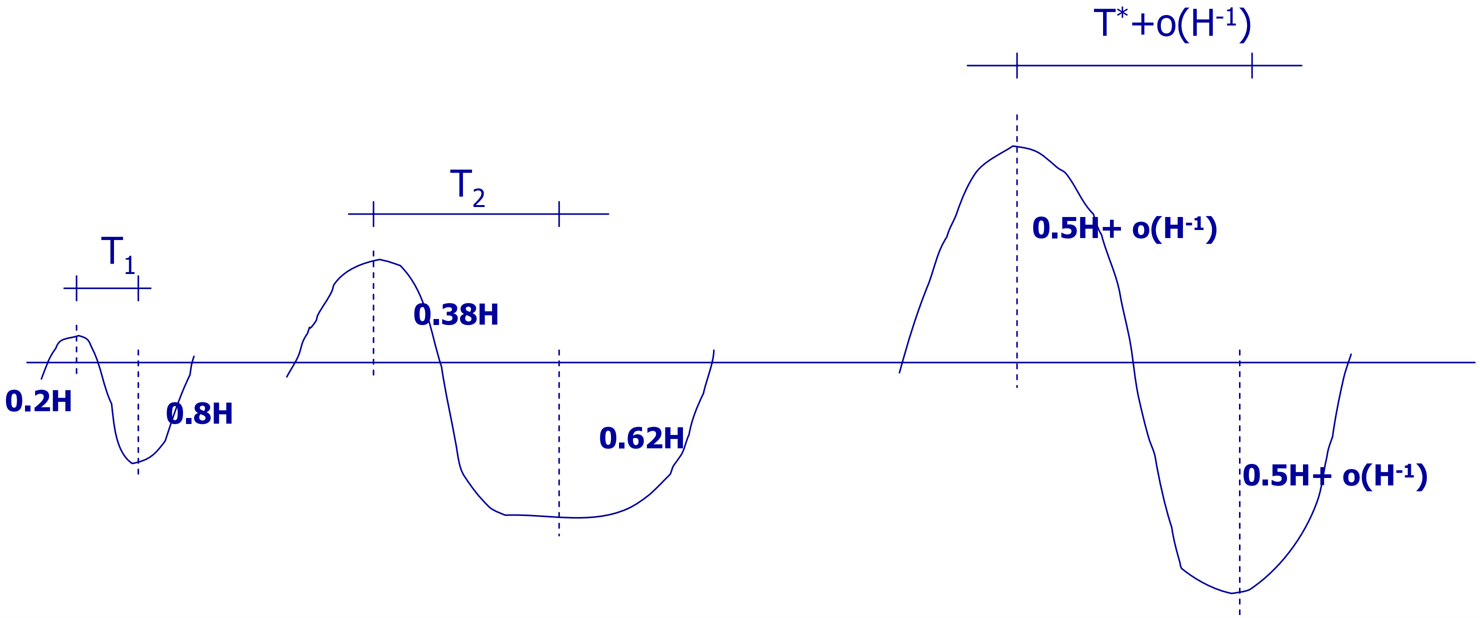
NECESSARY AND SUFFICIENT CONDITIONS TO HAVE A HIGH WAVE

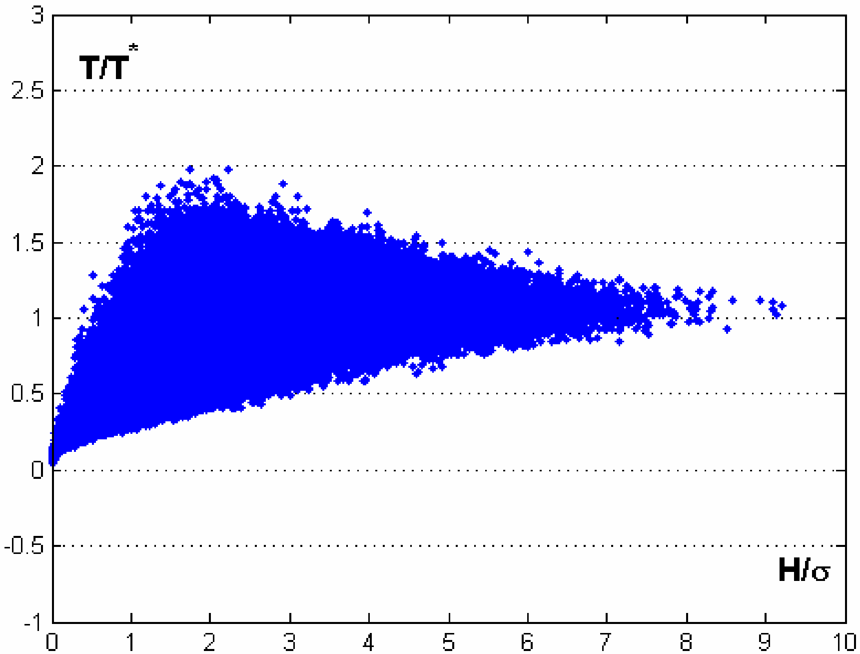


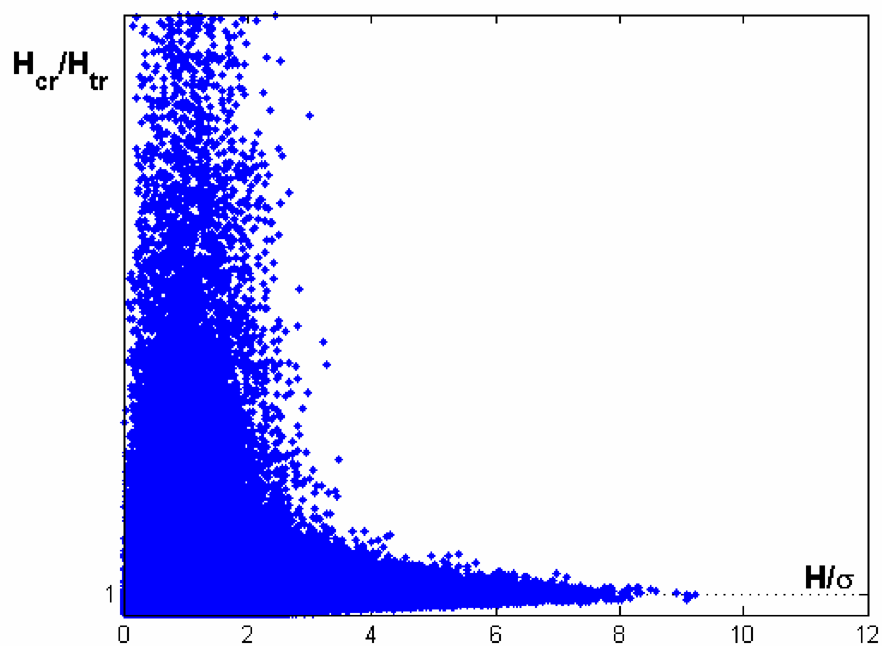
$$\eta(0) = \frac{H}{2} \qquad \eta(T^*) = -\frac{H}{2}$$



$$\psi(T) = \langle \eta(x_0, t_0)\eta(x_0, t_0 + T) \rangle$$


T1
0.2H
0.8H
T2
0.38H
0.62H
T*+o(H-1)
0.5H+ o(H-1)
0.5H+ o(H-1)

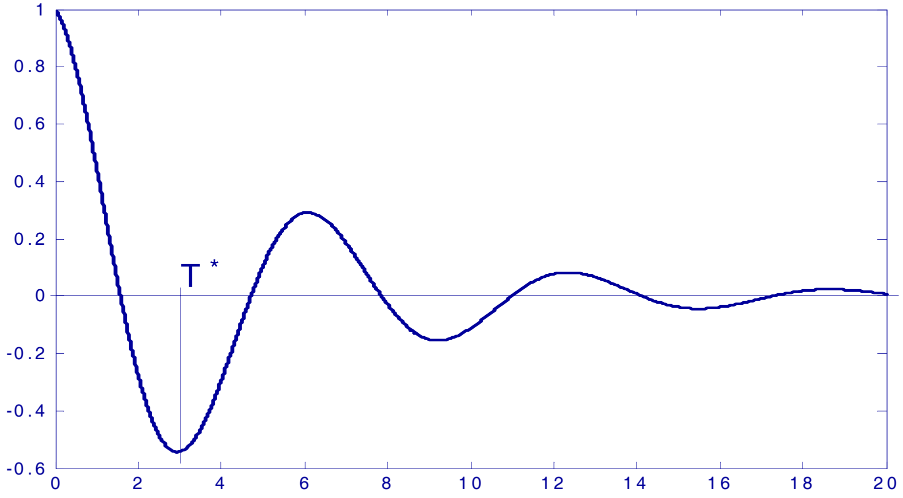

T/T*
H/σ
3
2.5
2
1.5
1
0.5
0
-0.5
-1
0 1 2 3 4 5 6 7 8 9 10


Hcr/Htr
H/σ
1
0 2 4 6 8 10 12

PROBABILITY OF EXCEEDANCE OF THE WAVE HEIGHT

Asymptotic expressions of Boccotti valid for any shape of sprectrum

$$P[H] = \exp\left[-\frac{1}{4(1+\psi^*)}\left(\frac{H}{\sigma}\right)^2\right] \quad per \quad \frac{H}{\sigma} \to \infty$$



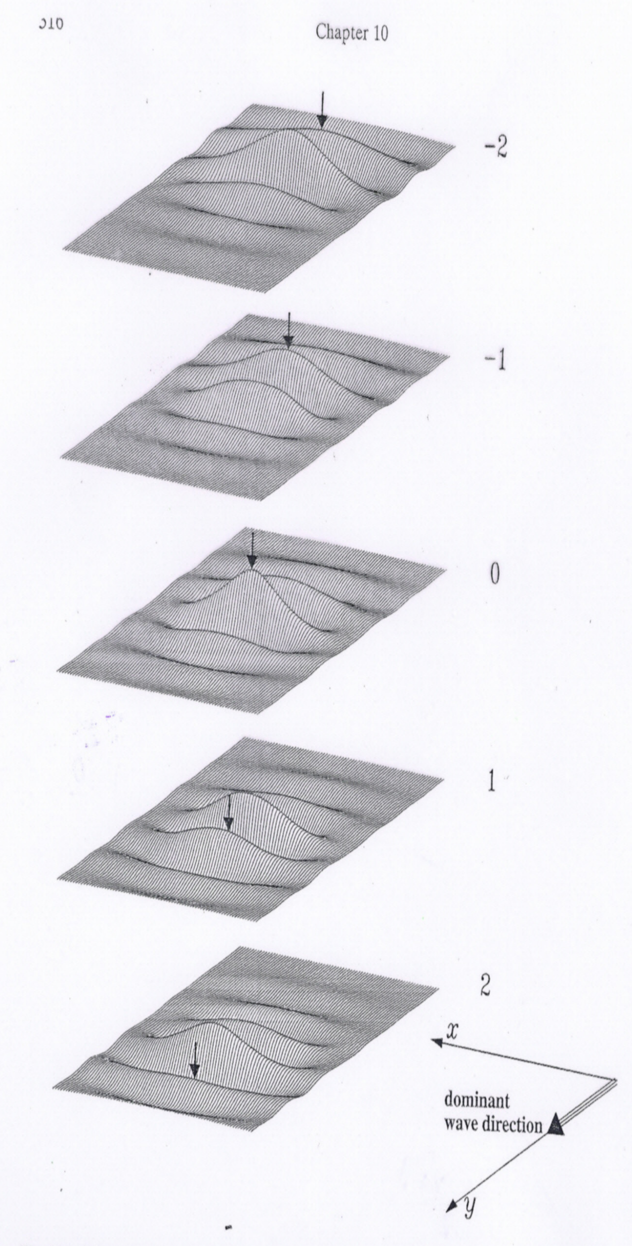
SPACE-TIME DOMAIN ANALYSIS

What happens in the neighborhood of a point $\mathbf{x}_0$ if a large crest followed by large trough are recorded in time at $\mathbf{x}_0$?

$$\Pr\left[\begin{array}{l}\eta(\mathrm{x}_0+\mathrm{X},t_0+T)\in\left(u,u+du\right)\\ \\ conditioned \quad to\\ \eta(\mathrm{x}_0,t_0)=H/2,\;\; \eta(\mathrm{x}_0,t_0+T_2^*)=-H/2\end{array}\right]$$

$$\Downarrow \quad \frac{H}{\sigma}\rightarrow\infty$$

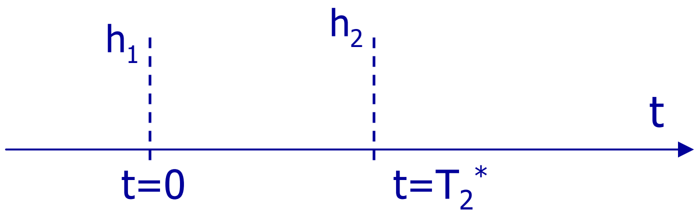
$$\delta\left(u-\eta_c\left(\mathbf{x}_0+\mathbf{X},t_0+T\right)\right)$$



TIME DOMAIN ANALYSIS : *SUCCESSIVE WAVE CRESTS*

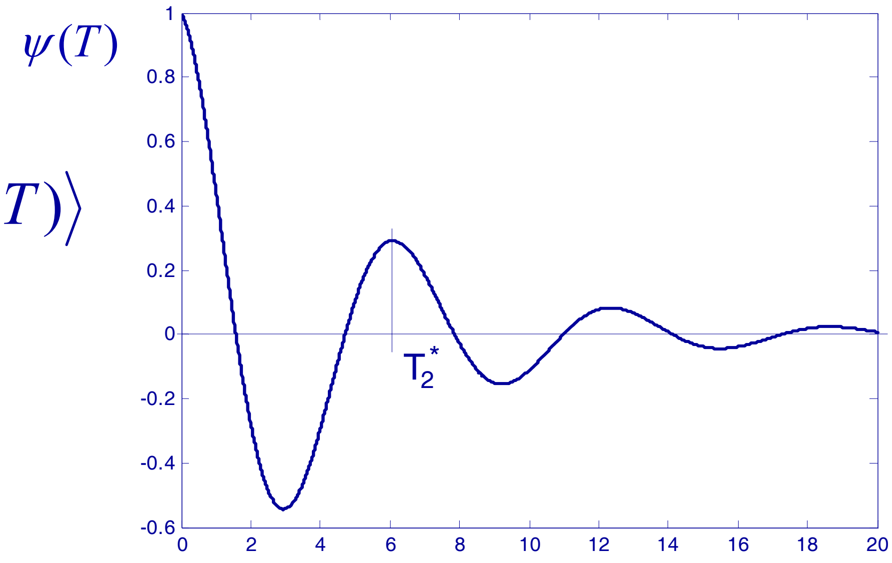
Necessary and sufficient conditions for the occurrence of two high wave crests

$$\eta(t_0) = h_1, \quad \eta(t_0 + T_2^*) = h_2$$



Autocovariance function

$$\psi(T) = \langle \eta(x_0, t_0)\eta(x_0, t_0 + T) \rangle$$

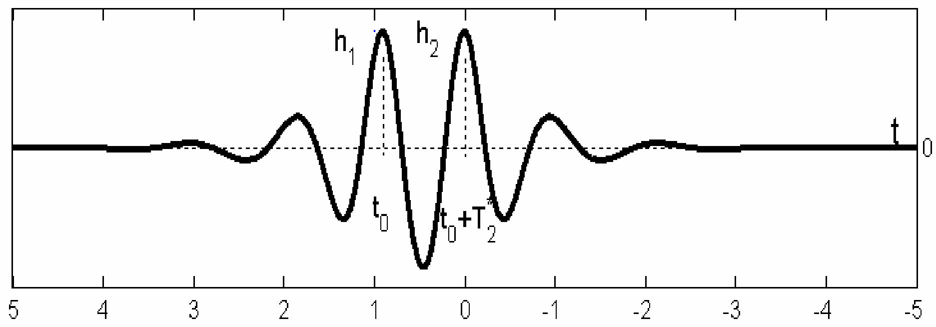


TIME DOMAIN : *THE CONDITIONS ARE SUFFICIENT*

$$\frac{h_1}{\sigma} \to \infty, \quad \frac{h_2}{\sigma} \to \infty$$

$$\Pr\left[\begin{matrix} \eta(t_0+T) \in (u, u+du) \\ conditioned\ to\ \ \eta(t_0)=h_1,\ \eta(t_0+T_2^*)=h_2 \end{matrix}\right] \quad \to \quad \delta\left[u-\eta_c(t_0+T)\right]$$

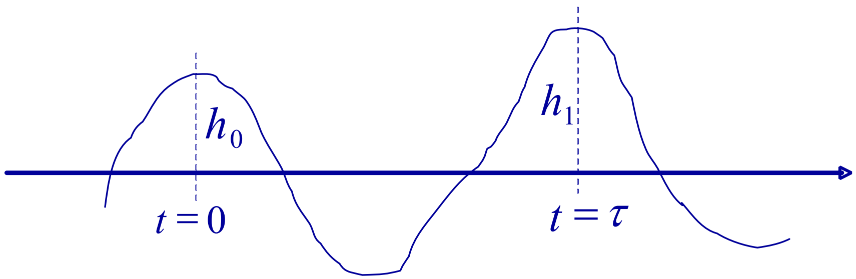
$$\eta_c(t_0+T) = \frac{h_1 - h_2\,\psi(T_2^*)/\psi(0)}{1-\left(\psi(T_2^*)/\psi(0)\right)^2}\,\psi(T) + \frac{h_2 - h_1\,\psi(T_2^*)/\psi(0)}{1-\left(\psi(T_2^*)/\psi(0)\right)^2}\,\psi(T-T_2^*)$$



TIME DOMAIN : *THE CONDITIONS ARE NECESSARY*

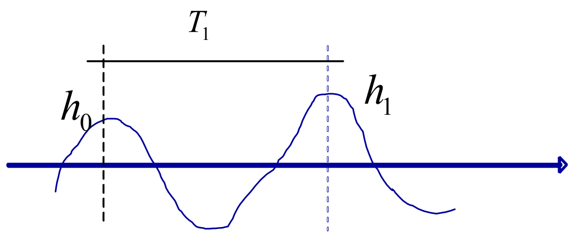
$$EX_c\left(h_1, h_2, \tau\right)$$

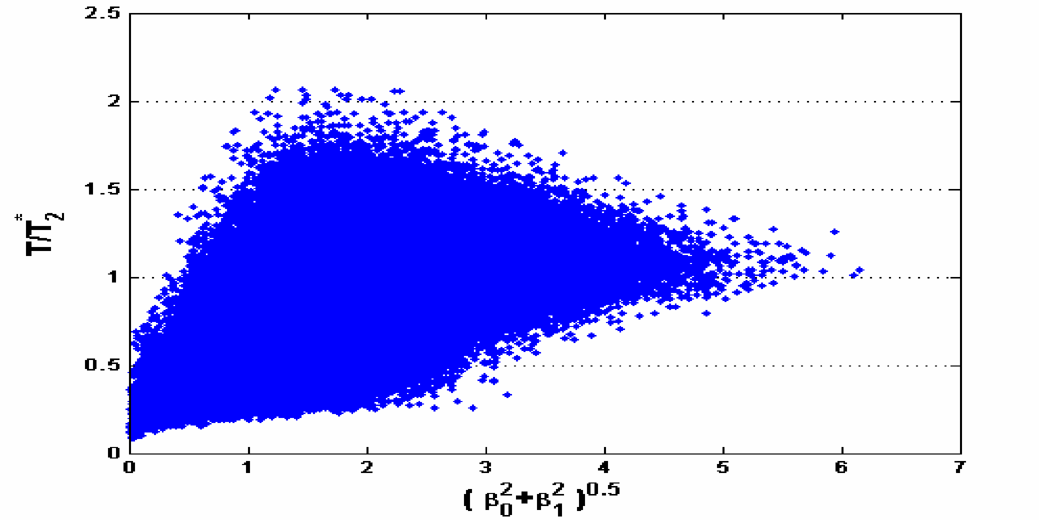
Expected number of local maxima of the surface displacement $\eta(t)$ **of amplitude** h_0 **which are followed by a local maximum with amplitude** h_1 **after a time lag** τ

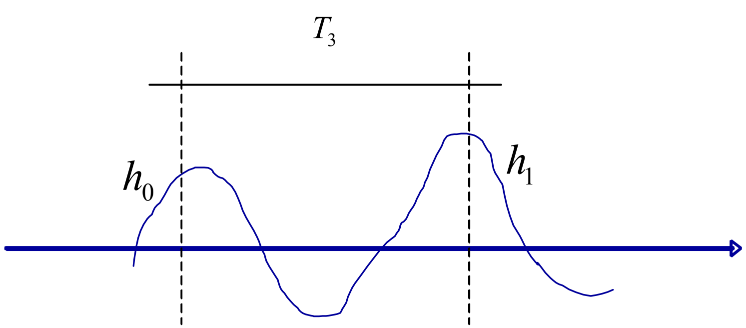


$$\beta_0 = \frac{h_0}{\sigma} \to \infty, \qquad \beta_1 = \frac{h_1}{\sigma} \to \infty$$

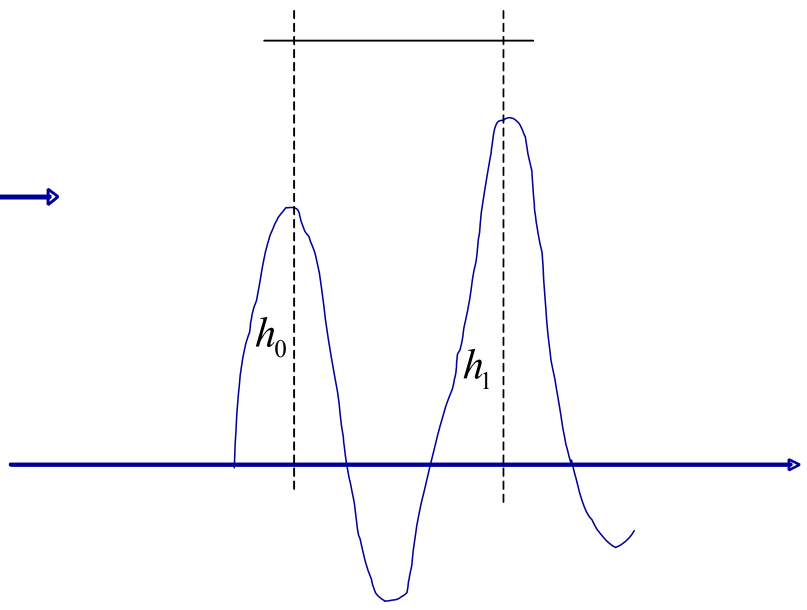
$$EX_{s.c.}\left(h_1, h_2, \tau\right) = \begin{cases} EX_c\left(h_1, h_2, T_2^*\right)\exp\left[-\frac{1}{2}K^*\delta\tau^2\right] & \left(\delta\tau\right) \propto O\left(\beta_0^{-1}, \beta_1^{-1}\right) \\ 0 & elsewhere \end{cases}$$







$$T_2^* + O\left(h_0^{-1}, h_1^{-1}\right)$$



$$as \quad h_0 \to \infty, \quad h_1 \to \infty$$

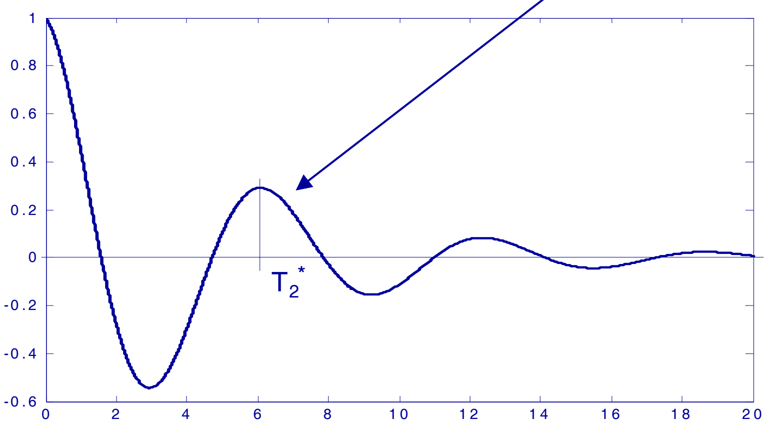
Corollary : joint probability of successive wave crests

$$p(\beta_0,\beta_1) \propto \int_0^\infty EX_c(\beta_0,\beta_1,\tau)\,d\tau$$

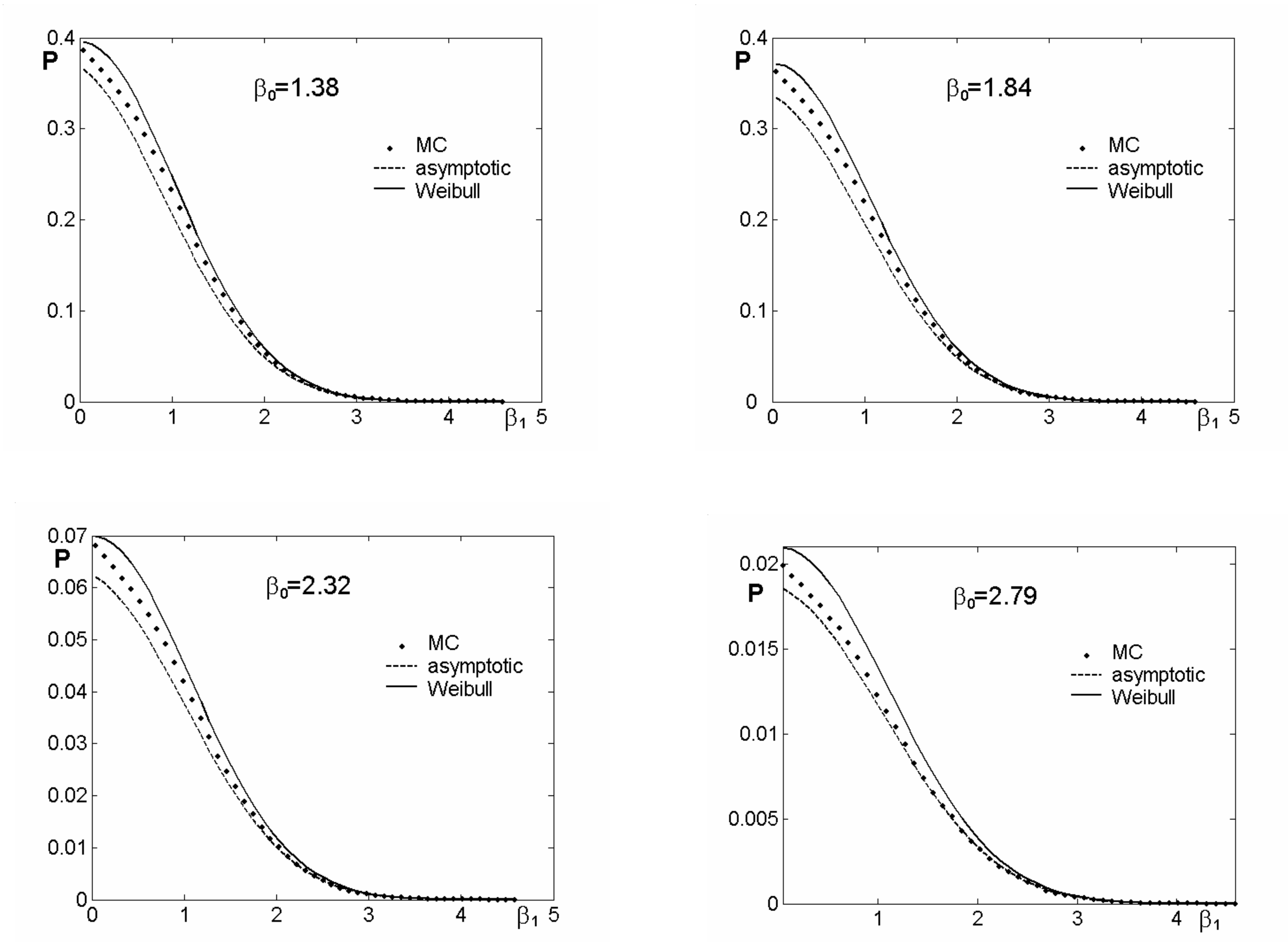
$$p(\beta_0,\beta_1) = \frac{1+\ddot{\psi}_2^*\psi_2^*}{\sqrt{-2\pi\ddot{\psi}_2^*\left(1-\psi_2^{*2}\right)^3}}\exp\left[-\frac{\beta_0^2+\beta_1^2-2\psi_2^*\beta_0\beta_1}{2\left(1-\psi_2^{*2}\right)}\right]\sqrt{\left(-\beta_0+s\beta_1\right)\left(-\beta_1+s\beta_2\right)}$$

$$p(\beta_0,\beta_1) = \frac{\beta_0\beta_1}{1-k^2}\exp\left[-\frac{\beta_0^2+\beta_1^2}{2\left(1-k^2\right)}\right]I_0\left(\frac{k\beta_0\beta_1}{1-k^2}\right)$$

Bivariate Weibull



MONTE CARLO SIMULATIONS OF GAUSSIAN SEAS



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$$\Downarrow \qquad \frac{h_1}{\sigma}\to\infty,\quad \frac{h_2}{\sigma}\to\infty$$

$$\delta\left(u-\eta_c\left(\mathbf{x}_0+\mathbf{X},t_0+T\right)\right)$$

$$\eta_c(\mathbf{X},T)=\frac{\psi(0)h_1-h_2\psi(T_2^*)}{\psi^2(0)-\psi(T_2^*)^2}\ \Psi(\mathbf{X},T)+\frac{\psi(0)h_2-h_1\psi(T_2^*)}{\psi^2(0)-\psi(T_2^*)^2}\ \Psi(\mathbf{X},T-T_2^*)$$

SPACE-TIME covariance

$$\Psi(\mathbf{X},T)=\left\langle\eta(x_0,t_0)\eta(x_0+\mathbf{X},t_0+T)\right\rangle$$

WAVE GROUP DYNAMICS