Successive Wave Crests in Gaussian Seas

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THEORY OF QUASI-DETERMINISM

necessary and sufficient conditions for the occurrence of an extreme wave

SUCCUSSIVE EXTREME WAVE CRESTS

necessary and sufficient conditions for their occurrence

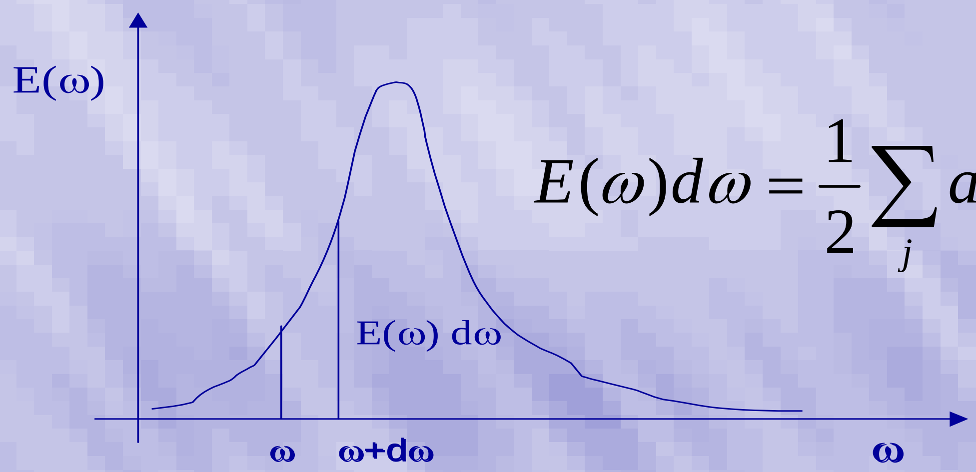
APPLICATIONS

Maximum Expected wave pressure in deep water

GAUSSIAN SEAS

$$\eta(t) = \sum_{j=1}^{N} a_j \cos(\omega_j t + \varepsilon_j)$$

ε_j *uniformly random in* $[0, 2\pi]$



$$E(\omega)d\omega = \frac{1}{2}\sum_j a_j^2 \qquad \omega < \omega_j < \omega + d\omega$$

Stationarity

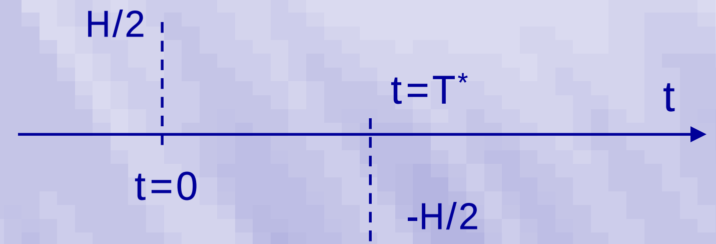
Ergodicity

Gaussianity

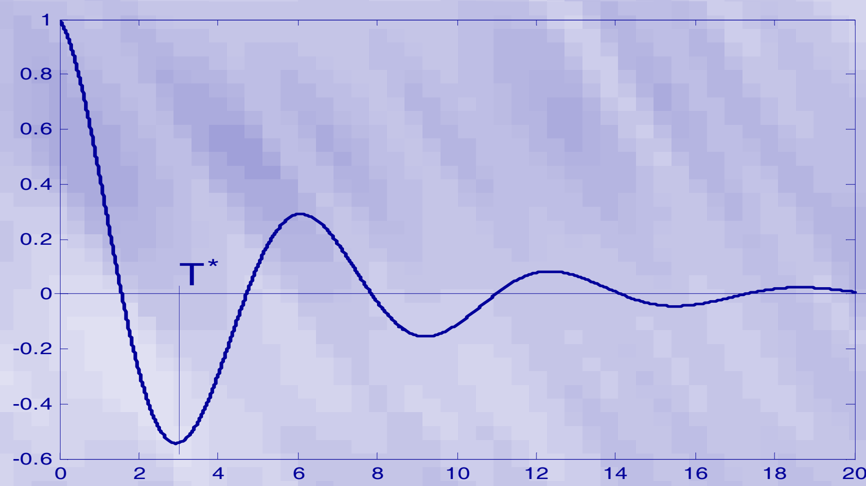
THEORY OF QUASI-DETERMINISM*

Necessary and sufficient conditions for the occurrence of an high wave crest

$$\eta(0) = \frac{H}{2} \qquad \eta\left(T^*\right) = -\frac{H}{2}$$



Autocovariance function $\psi(T)$

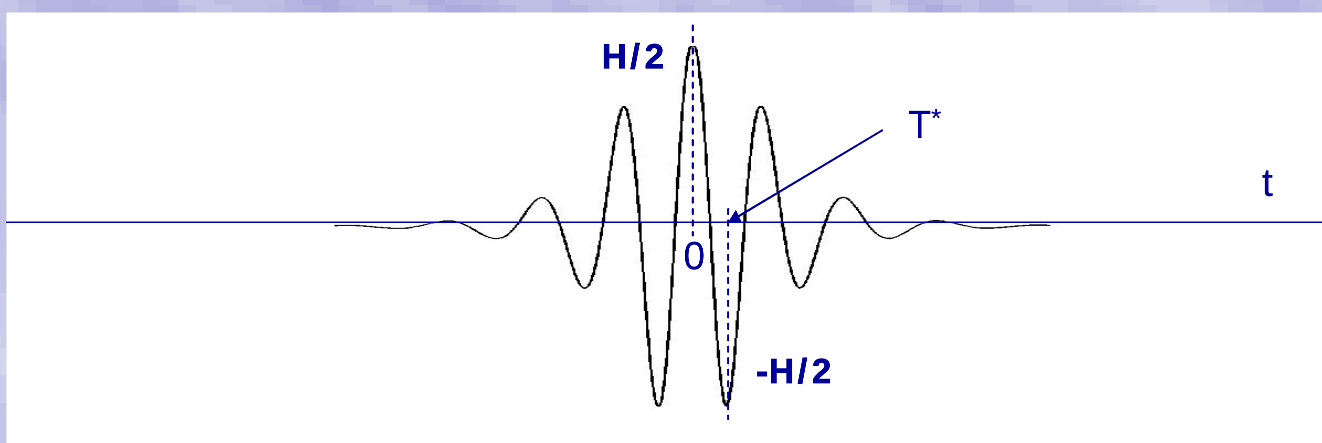


* Boccotti P. *Wave mechanics for ocean engineering* Elsevier Science 2000, Oxford

The conditions are sufficient

$$\Pr\left[\eta(t)=\eta/\eta(0)=\frac{H}{2},\eta(T^*)=-\frac{H}{2}\right] \xrightarrow{\frac{H}{\sigma}\to\infty} \delta[u-\bar{\eta}(t_0+T)]$$

$$\bar{\eta}(t_0+T) = \frac{H}{2}\,\frac{\psi(T)-\psi(T-T^*)}{\psi(0)-\psi(T^*)}$$

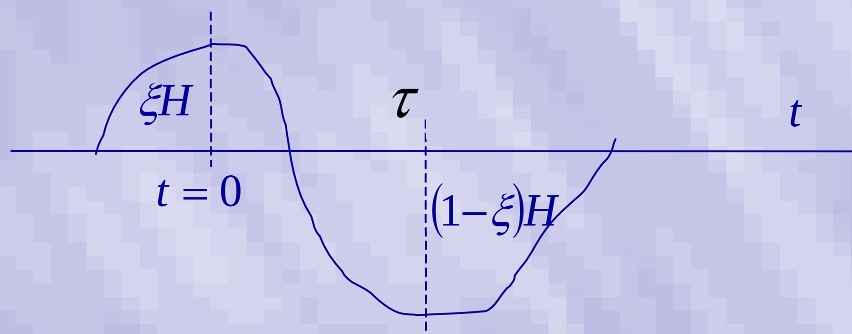


The conditions are necessary

$$EX(H,\tau,\xi)$$

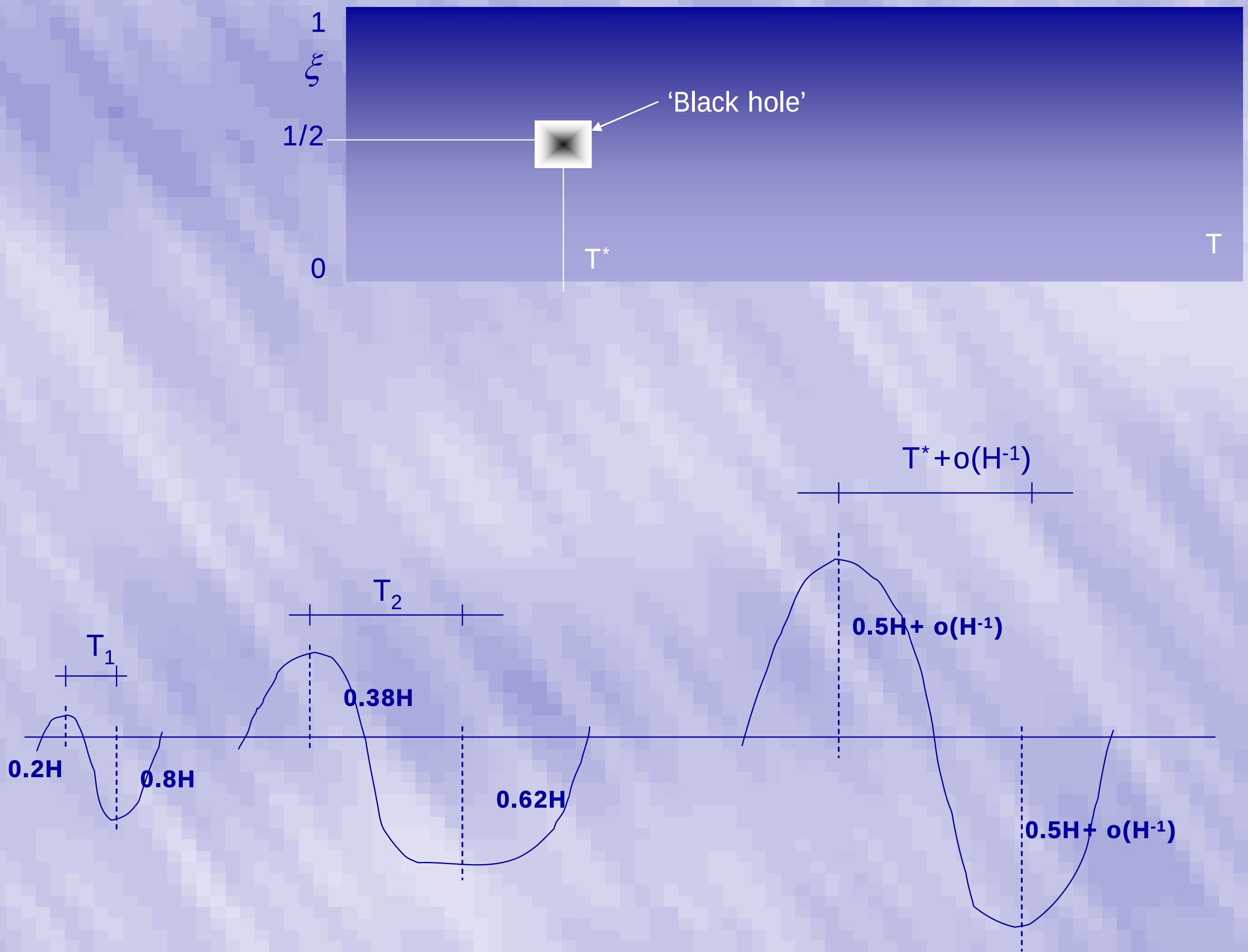
expected number of local maxima of the surface displacement $\eta(t)$ with amplitude ξH

which are followed by a local minimum with amplitude $(\xi - 1)H$ after a time lag τ



$$\alpha = H/\sigma \to \infty$$

$$EX(H,\tau,\xi) = \begin{cases} EX\left(H, T^*, \tfrac{1}{2}\right) \exp\left[-\tfrac{1}{8}\left(K_\tau^* \delta\tau^2 + K_\xi^* \delta\xi^2\right)\alpha^2\right] & (\delta\tau, \delta\xi) \sim O(\alpha^{-1}) \\ 0 & elsewhere \end{cases}$$

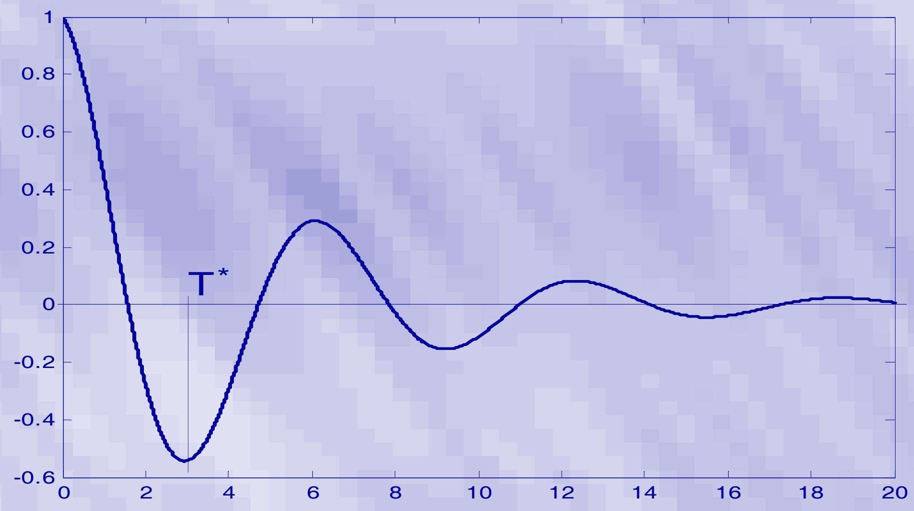


1
ξ
1/2
0
'Black hole'
T*
T
T*+o(H-1)
T2
T1
0.2H
0.8H
0.38H
0.62H
0.5H+ o(H-1)
0.5H+ o(H-1)

Corollary : probability of exceedance of wave height

Asymptotic expressions of Boccotti valid for any shape of sprectrum

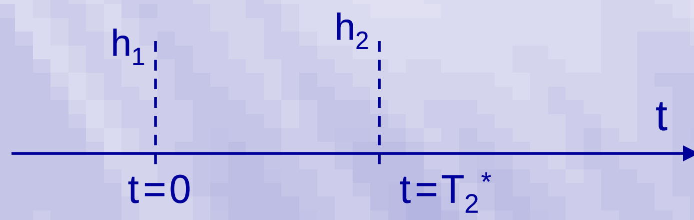
$$P[H]=\exp\left[-\frac{1}{4(1+\psi^*)}\left(\frac{H}{\sigma}\right)^2\right] \quad per \quad \frac{H}{\sigma}\rightarrow\infty$$



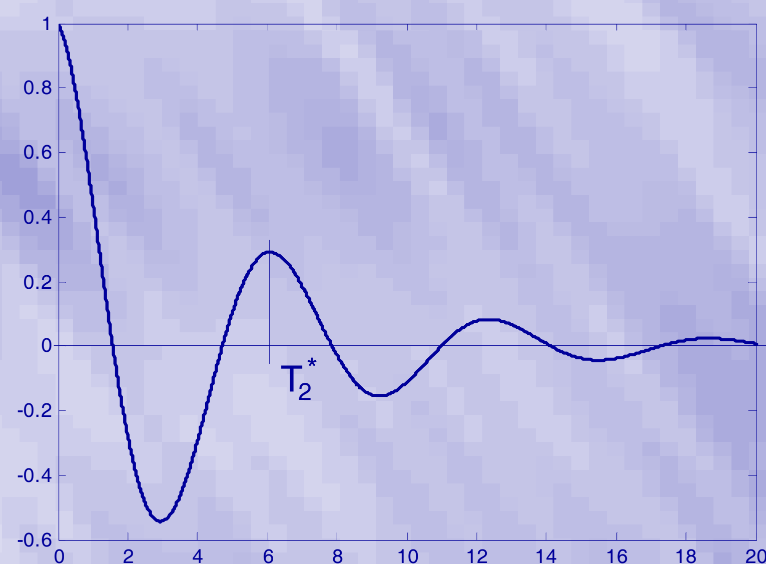
Successive Wave crests in gaussian seas

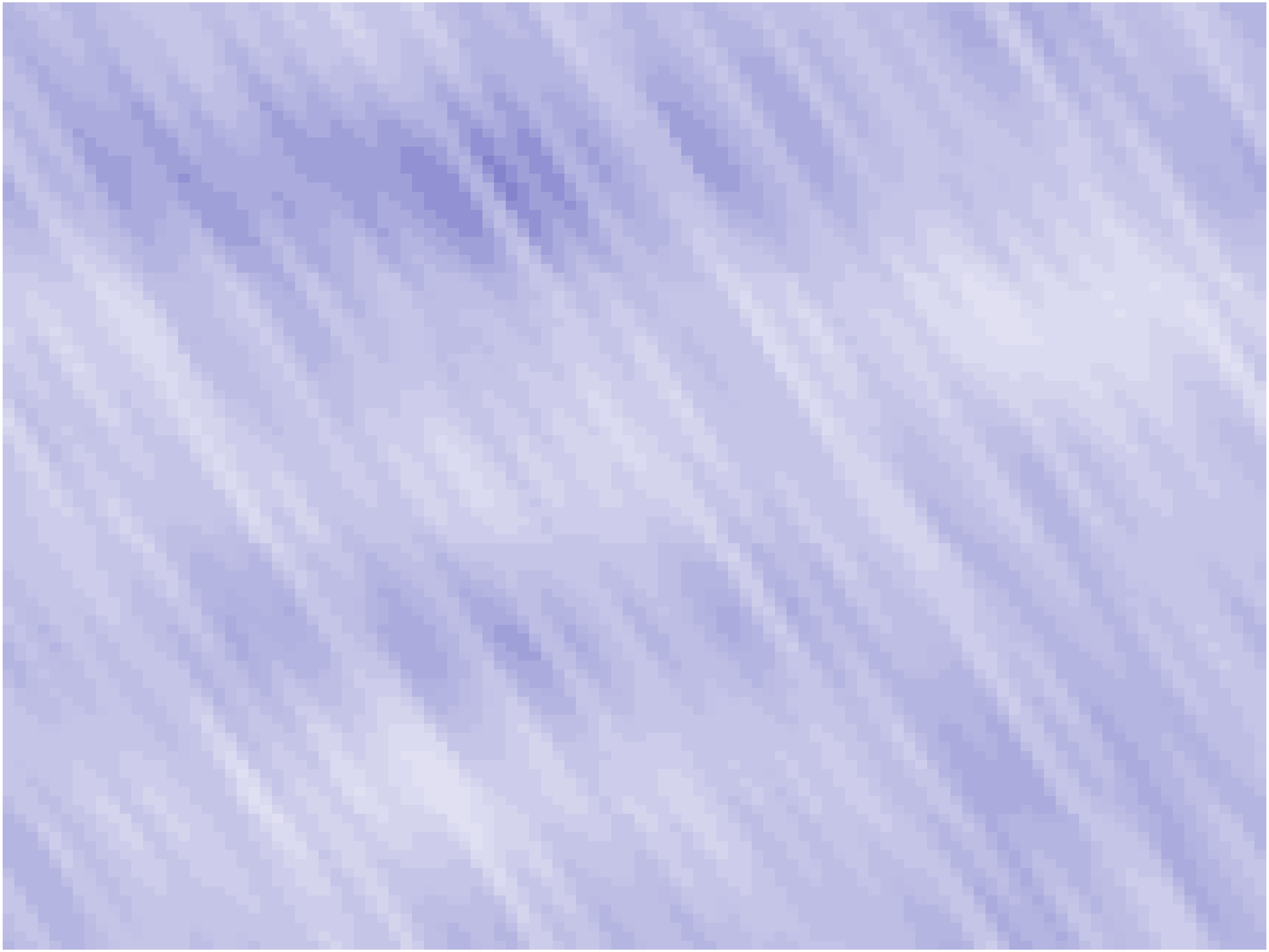
Necessary and sufficient conditions for the occurrence of two high wave crests

$$\eta(0) = h_1 \qquad \eta(T_2^*) = h_2$$



Autocovariance function $\psi(T)$



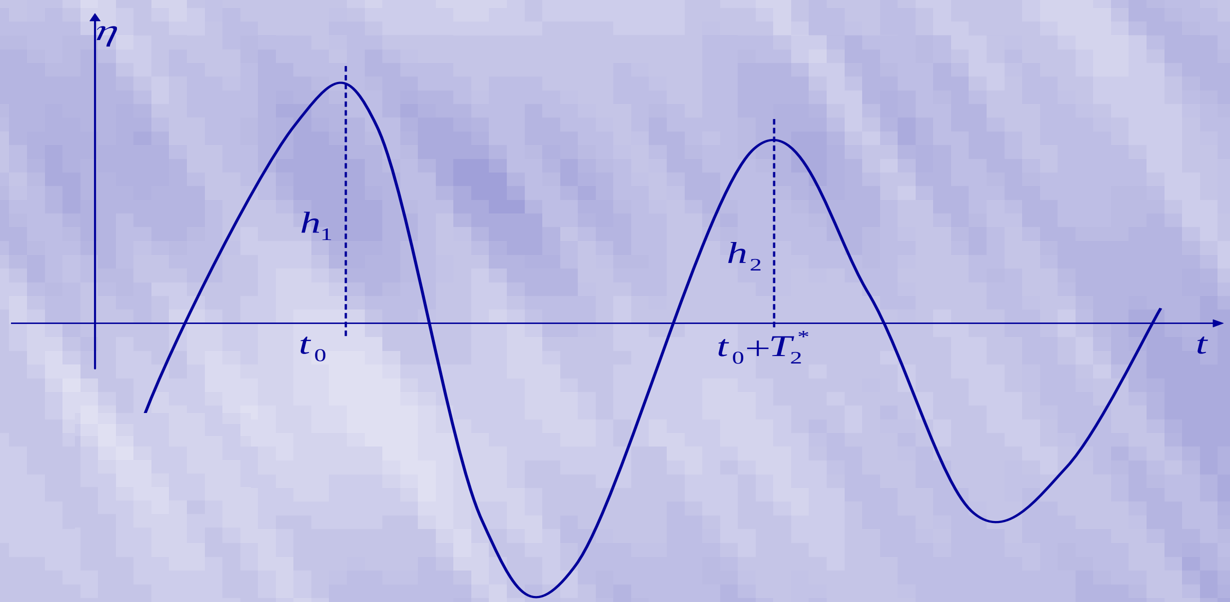


The conditions are sufficient

$$\Pr[\eta(t_0 + T) = u / \eta(t_0) = h_1, \eta(t_0 + T_2^*) = h_2]$$

$$\delta[u - \eta_c(t_0 + T)] \quad as \quad \frac{h_1}{\sigma} \quad \text{and} \quad \frac{h_2}{\sigma} \to \infty$$

$$\eta_c(t_0 + T) = \frac{h_1\psi(0) - h_2\psi(T_2^*)}{\psi^2(0) - \psi^2(T_2^*)}\psi(T) + \frac{h_2\psi(0) - h_1\psi(T_2^*)}{\psi^2(0) - \psi^2(T_2^*)}\psi(T - T_2^*)$$

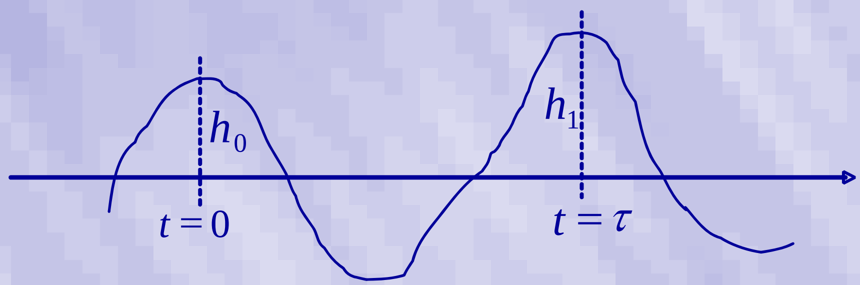


The conditions are necessary

$$EX_{s.c.}(h_0, h_1, \tau)$$

Expected number per unit time of local maxima of the surface displacement $\eta(t)$ with amplitude h_0

which are followed by a local maximum of amplitude h_1 after a time lag τ



$$\beta_0 = h_0/\sigma \to \infty \text{ and } \beta_1 = h_1/\sigma \to \infty$$

$$EX_c(h_0, h_1, \tau) = \begin{cases} EX_c(h_0, h_1, T_2^*) \exp\left(-\frac{1}{2} K^* \delta\tau^2\right) \\ 0 \quad elsewhere \end{cases}$$

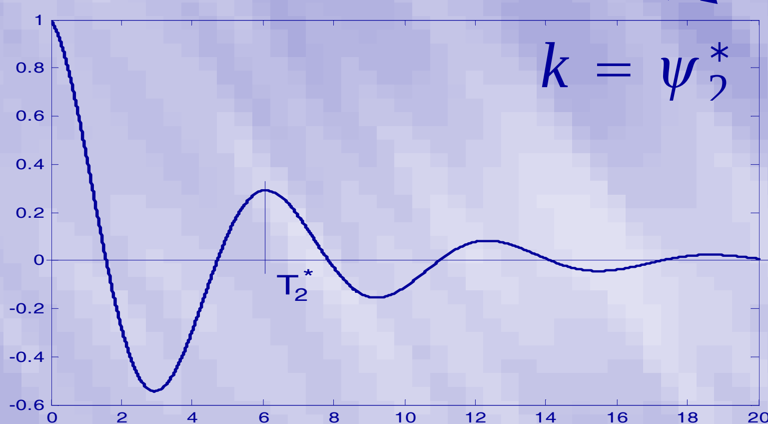
Corollary : joint probability of two successive crests

$$p(\beta_0,\beta_1) \sim \int_0^\infty EX_c(\beta_0,\beta_1,\tau)\,d\tau$$

$$p(\beta_0,\beta_1) = \frac{1+\psi_2^*\ddot{\psi}_2^*}{\sqrt{-2\pi\ddot{\psi}_2^*(1-\psi_2^{*2})^3}}\exp\left[-\frac{\beta_0^2+\beta_1^2-2\psi_2^*\beta_0\beta_1}{2(1-\psi_2^{*2})}\right]\sqrt{(-\beta_0+s\;\beta_1)(-\beta_1+s\;\beta_0)}$$

$$p_W(\beta_0,\beta_1) = \frac{\beta_0\beta_1}{(1-k^2)}\exp\left[-\frac{\beta_0^2+\beta_1^2}{2(1-k^2)}\right]I_0\left(\frac{k\;\beta_0\beta_1}{1-k^2}\right)$$

Bivariate Weibull

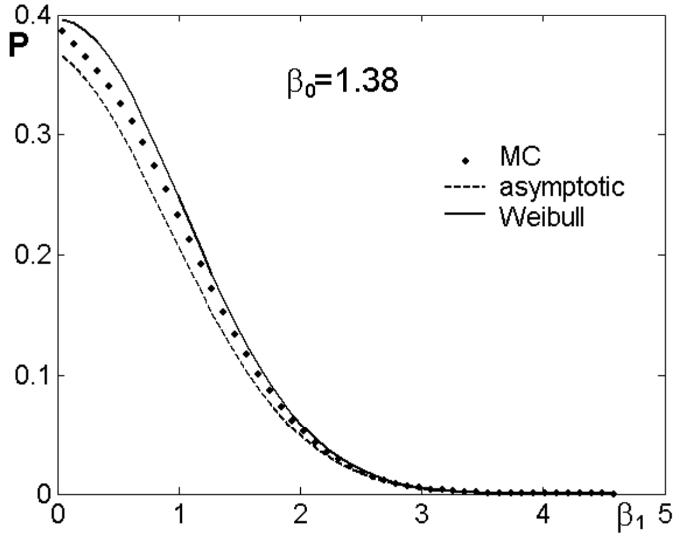


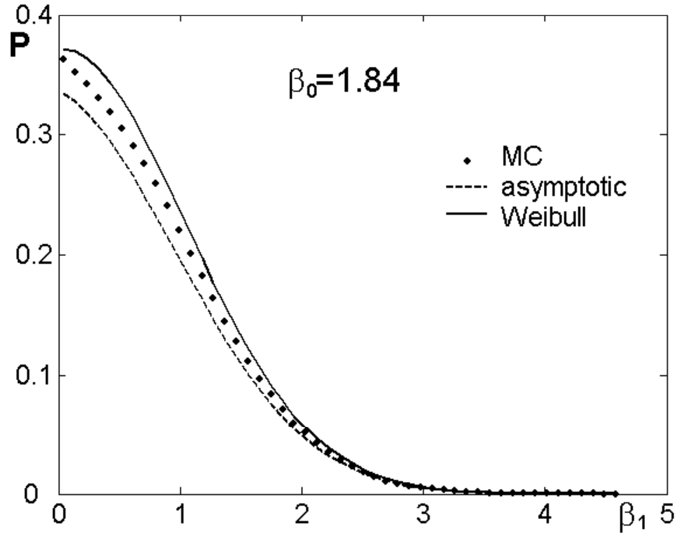
$$k = \psi_2^*$$

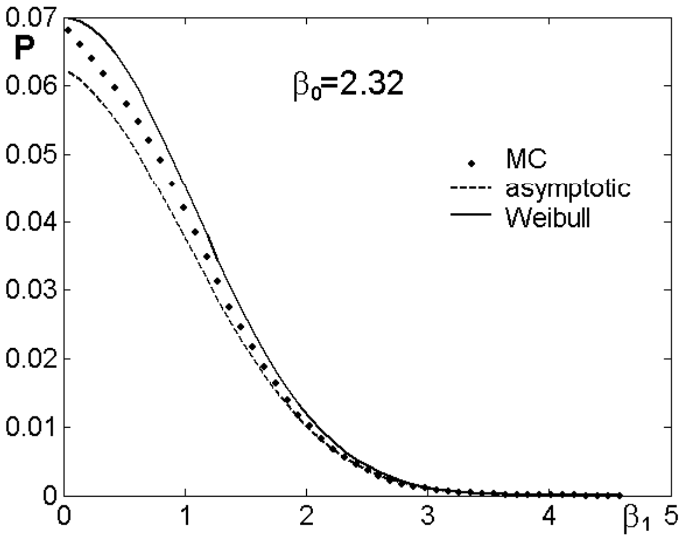
In Ocean applications

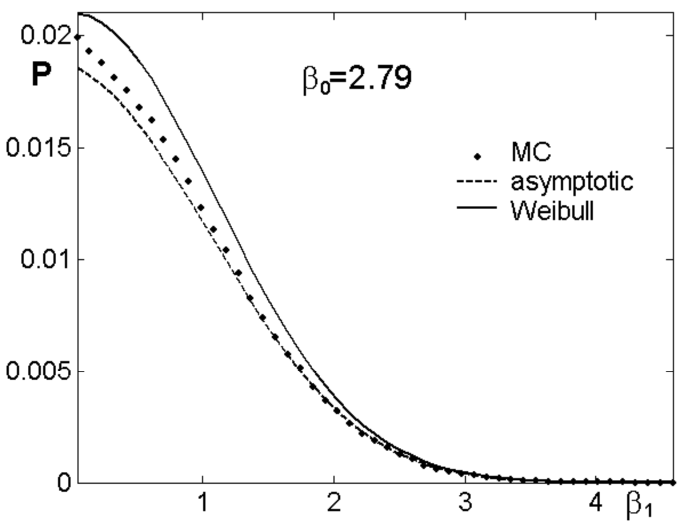
$$k_m = \sqrt{\frac{\left[\int_0^\infty S(\omega)\cos(\omega T_m)\,d\omega\right]^2+\left[\int_0^\infty S(\omega)\sin(\omega T_m)\,d\omega\right]^2}{\sigma^2}}$$

Monte Carlo Simulations









Maximum Expected wave pressure in deep water

$$\Pr(C_{\max} \le h) = \Pr(C_1 \le h, C_2 \le h, \ldots\ldots, C_{N_c} \le h)$$

Stochastic independence

$$[\Pr(C_1 \le h)]^{N_c}$$

Stochastic dependence

$$\Pr(C_1 \le h) \bullet [\Pr(C_j \le h / C_{j-1} \le h)]^{N_c - 1}$$